

# Solutions Manual

to accompany

# Introduction to Engineering Design and Problem Solving

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Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis  
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# INTRODUCTION TO ENGINEERING DESIGN PORTFOLIO

**Name**

**Section**

## **Problem Statement**

**In your own words, describe the problem clearly so someone else can understand it. What will the solution accomplish? Are there imposed specifications and limitations?**

## **Investigation**

What are some questions that must be answered to solve the problem? List at least three.

## **Resources**

Make a list of resources that you have used to obtain information about the problem to answer questions you listed above. These may include people, written material or electronic media.

## **Information**

What information have you gathered from the resources noted above?

1. Information from people.
2. Information from written material.
3. Information from electronic media.

## **Brainstorming for Ideas**

**Sketching is a great way to generate ideas. Use the space below to draw or sketch as many ideas as you can think of. At this point, do not eliminate anything that may have possibilities. You may want to add additional sketches, perhaps using graph paper.**

## **Alternative Solutions--Describe Your Best Ideas**

**Describe three of your most workable solutions to the problem. Remember to consider the specifications and limitations. In your description indicate what you consider each solution's strengths and weaknesses.**

**Solution 1.**

**Solution 2.**

**Solution 3.**

## Selecting Your Best Solution

**Describe your best solution and indicate below why you selected this solution.**

**Describe how you are going to construct the solution to the problem.**

## **Construct Your Solution**

**Construct your solution to the problem based on the specifications and limitations. Describe any modifications to the design you made as a result of the construction process. How did these alter the solution?**

## **Analysis and Testing**

**Describe how you will test the design to determine if it works. Plan to conduct the test more than once to insure that the results are repeatable and not just luck.**

**Describe the results of your tests below. Use graphs, charts as appropriate supplements.**



## **Communicating to the Class**

**If you are communicating your solution to the class, what media will you use?**

**Outline the contents of your presentation below.**

## Design Assessment Rubrics

### The Design Process

A. Identified problem criteria, constraints and specifications	0	1	2	3
B. Gathered background information from a variety of sources	0	1	2	3
C. Suggested several alternative solutions	0	1	2	3
D. Evaluated ideas against design criteria and made improvements	0	1	2	3
E. Justified the chosen solution	0	1	2	3

### The Design Solution

A. Provided an accurate drawing with basic details and dimensions	0	1	2	3
B. Constructed the model and used materials appropriately	0	1	2	3
C. The solution worked. It fulfilled the design criteria.	0	1	2	3
D. Originality and creativity of the design	0	1	2	3

### Testing

A. Used knowledge gained from testing to inform design	0	1	2	3
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### Work Habits

A. Completed assigned task in a timely fashion	0	1	2	3
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### Communication and Presentation

A. Demonstrated understanding of key ideas orally and/or in writing	0	1	2	3
B. Report neatly written with good grammar	0	1	2	3

**Scoring Guide:** 0 = No response or unacceptable response

1 = Acceptable response

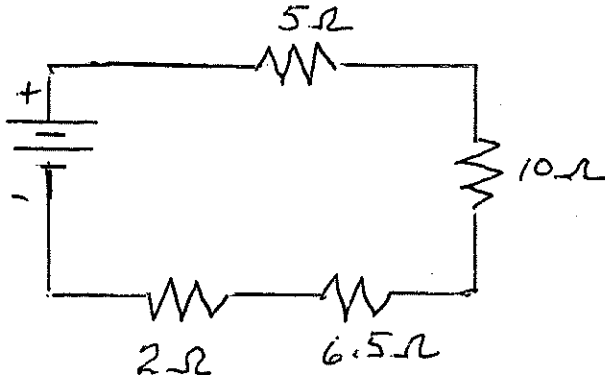
2 = Good response

3 = Excellent response

Score \_\_\_\_\_  
Total possible points 39

## Chapter Four

4.1 Find the equivalent resistance for the circuit in Figure P4.1.



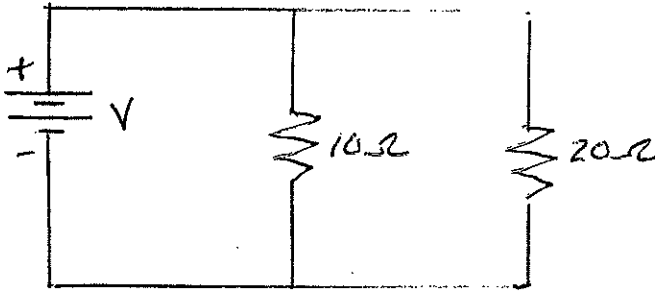
Series Circuit

$$R_{eq} = \sum_i R_i$$

$$R_{eq} = 5 + 10 + 2 + 6.5$$

$$R_{eq} = 23.5\Omega$$

4.2 Find the equivalent resistance for the circuit in Figure P4.2.



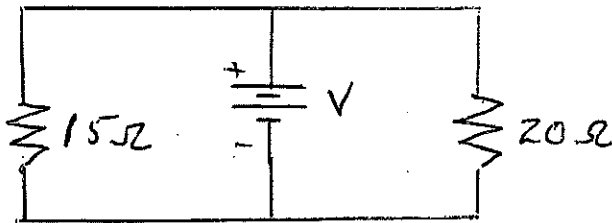
Parallel Circuit

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{20} = 0.15$$

$$R_{eq} = 6.67\Omega$$

4.3 Find the equivalent resistance for the circuit in Figure P4.3.



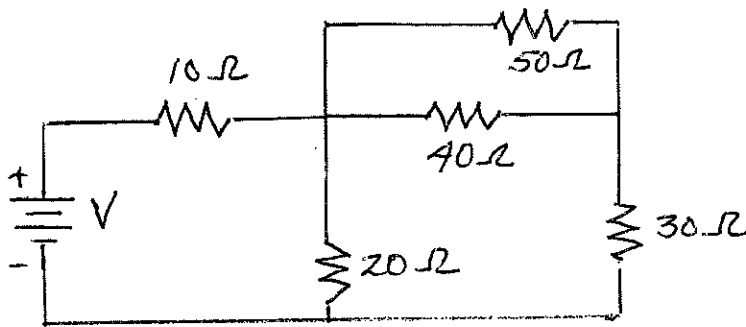
Parallel Circuit

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

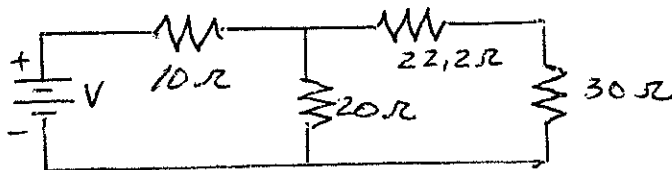
$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{15} = 0.116$$

$$R_{eq} = 8.57\Omega$$

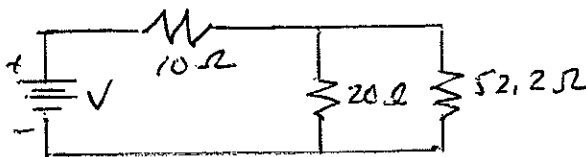
4.4 Find the equivalent resistance for the circuit in Figure P4.4.



$$\frac{1}{R_{eq}} = \frac{1}{50} + \frac{1}{40} = 0.045 \quad R_{eq} = 22.2 \Omega$$

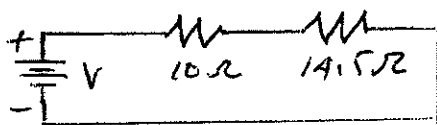


$$R_{eq} = 22.2 + 30 = 52.2 \Omega$$



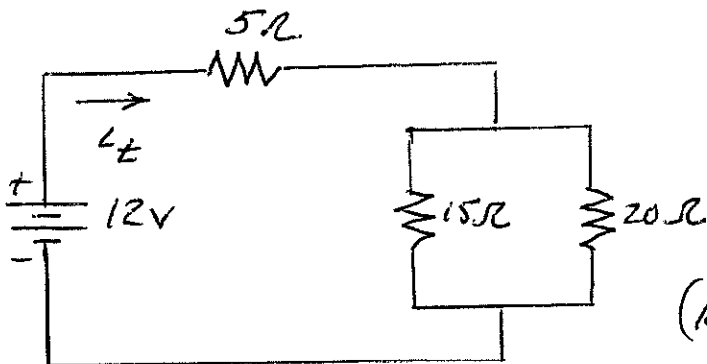
$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{52.2} = 0.069$$

$$R_{eq} = 14.5 \Omega$$



$$R_{eq} = 10 + 14.5 = 24.5 \Omega$$

4.5 Find the equivalent resistance for the circuit in Figure P4.5 and the current flow through each resistor.



$$\frac{1}{R_{eq}} = \frac{1}{15} + \frac{1}{20}$$

$$R_{eq} = 8.57 \Omega$$

$$(R_{eq})_{circuit} = 5 + 8.57 = 13.57 \Omega$$

$$I_t = \frac{12}{13.57} = 0.884 \text{ amp}$$

$$I_{5\Omega} = 0.884 \text{ amp}$$

Kirchhoff's law  $V \uparrow = V \downarrow$

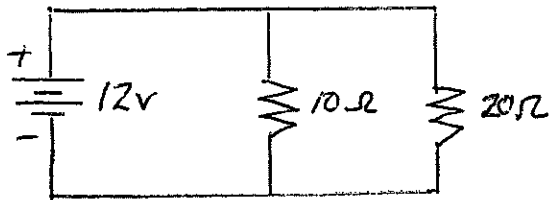
$$12 = (0.884 \times 5) + V_{\text{parallel}}$$

$$V_{\text{parallel}} = 7.58 \text{ v}$$

$$i = \frac{V}{R} \quad i_{15\Omega} = \frac{7.58}{15} = 0.505 \text{ amp}$$

$$i_{20\Omega} = \frac{7.58}{20} = 0.379 \text{ amp}$$

4.6 Find the current flow through each resistor in Figure P4.2 if the voltage source is 12 V.

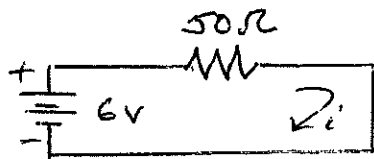


$$i = \frac{V}{R}$$

$$i_{10} = \frac{12}{10} = 1.2 \text{ amp}$$

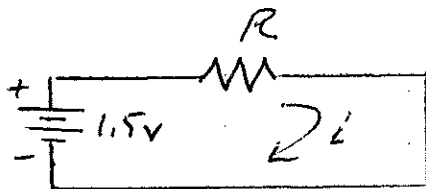
$$i_{20} = \frac{12}{20} = 0.6 \text{ amp}$$

4.7 The filament in a flashlight bulb has a resistance of 50 Ω. The battery voltage is 6 volts, determine the current flow.



$$i = \frac{V}{R} = \frac{6}{50} = 0.12 \text{ amp}$$

4.8 The maximum current flow from a 1.5 volt battery is 45 mA. What is the minimum size resistor that can be connected to it?

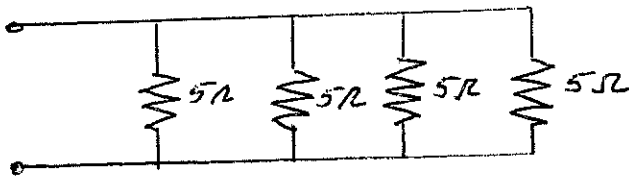


$$i = 45 \text{ mA} = 0.045 \text{ A}$$

$$V = iR$$

$$R = \frac{1.5}{0.045} = 33.3 \Omega$$

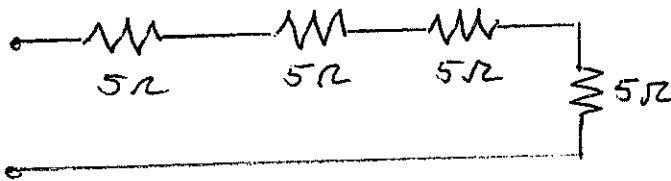
4.9 Four  $5\ \Omega$  resistors are wired in parallel circuit, what is the equivalent resistance? If they were wired in series, what is the equivalent circuit resistance?



$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

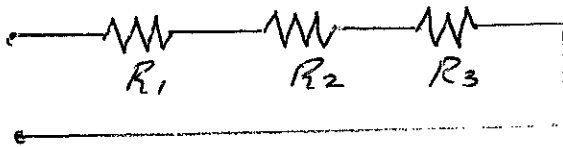
$$R_{eq} = 1.25\ \Omega$$



$$R_{eq} = \sum_i R_i$$

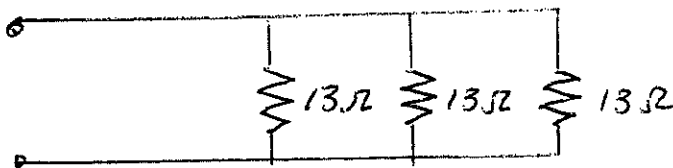
$$R_{eq} = 5 + 5 + 5 + 5 = 20\ \Omega$$

4.10 The equivalent resistance of a three-resistor series circuit is 39 ohms. If the three resistors, each of the same value, are now connected in parallel, what is the equivalent circuit resistance?



$$R_{eq} = \sum_i R_i = 3R$$

$$R = \frac{39}{3} = 13\ \Omega$$

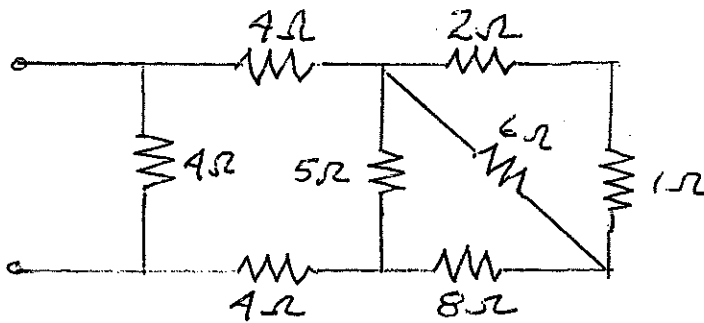


$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_{eq}} = \frac{1}{13} + \frac{1}{13} + \frac{1}{13}$$

$$R_{eq} = 4.33\ \Omega$$

4.11 Find the equivalent resistance for the circuit shown in Figure P4.11.

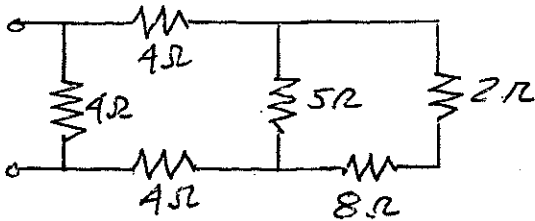


Combine  $1\Omega + 2\Omega$   
in series

$$R_{eq} = 3\Omega$$

This acts in parallel  
to  $6\Omega$  resistor

$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}; R_{eq} = 2\Omega$$

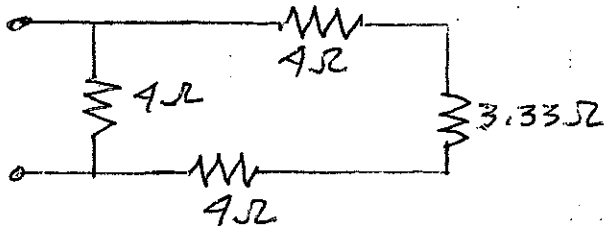


The  $2\Omega + 8\Omega$  are in  
series;  $R_{eq} = 10\Omega$ .

This is in parallel with the  
 $5\Omega$  resistor.

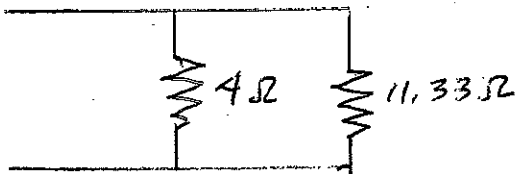
$$\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

$$R_{eq} = 3.33\Omega$$



$$(R_{eq})_{series} = 4 + 3.33 + 4$$

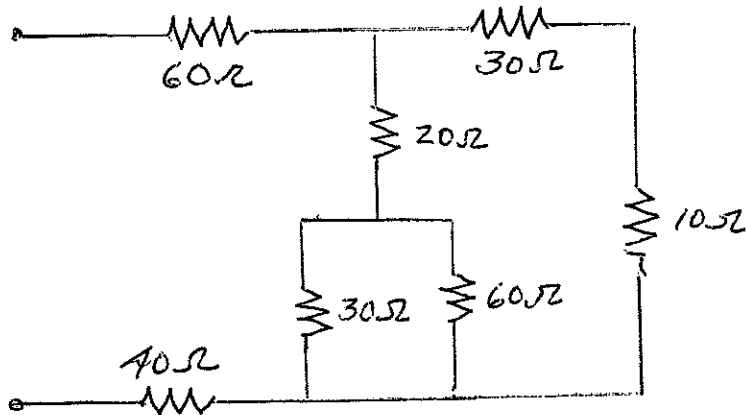
$$R_{eq} = 11.33\Omega$$



$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{11.33}$$

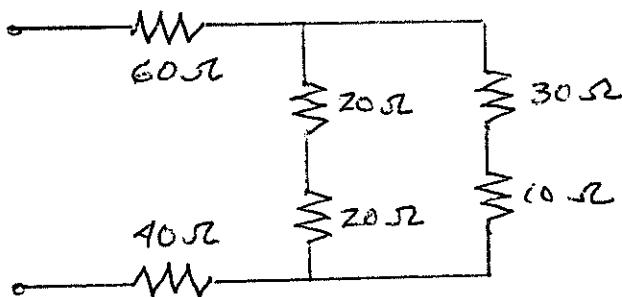
$$R_{eq} = 2.96\Omega$$

4.12 Find the equivalent resistance for the circuit shown in Figure P4.12.



Find  $R_{eq}$  of parallel  $30\Omega + 60\Omega$  resistances

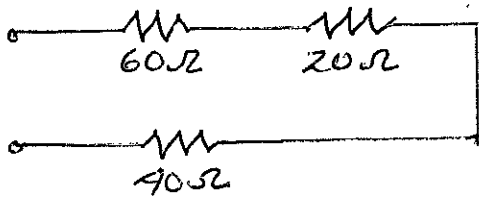
$$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{60} = 0,05 \quad R_{eq} = 20\Omega$$



Determine  $R_{eq}$  of  
The parallel circuit

$$\frac{1}{R_{eq}} = \frac{1}{40} + \frac{1}{40} = 0,05$$

$$R_{eq} = 20\Omega$$

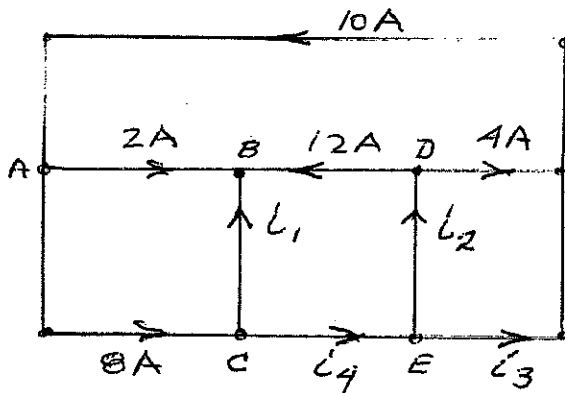


$$R_{eq} = \sum_i R_i$$

$$R_{eq} = 60 + 20 + 40 = \underline{120\Omega}$$



4.13 Use Kirchhoff's current law to determine the unknown currents in Figure P4.13.



Kirchhoff Current Law  
 $\sum i_{in} = \sum i_{out}$   
 or algebraic sum of currents at a node is zero.

Node A  $10 - 2 - 8 = 0$

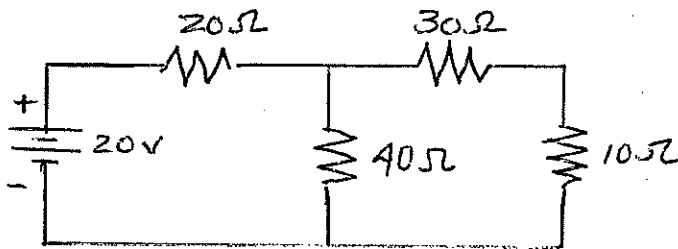
Node B  $i_1 + 2 + 12 = 0$   $i_1 = -14A$  in opposite direction

Node C  $8 + 14 - i_4 = 0$   $i_4 = 22A$

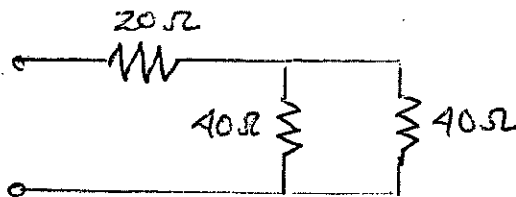
Node D  $-12 + i_2 - 4 = 0$   $i_2 = 16A$

Node E  $22 - 16 - i_3 = 0$   $i_3 = +6A$

4.14 Find the power absorbed in the  $10\ \Omega$  resistor in Figure P4.14.

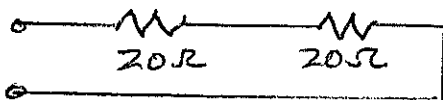


Find The  $R_{eq}$  for The circuit and then current flows to determine  $i^2 R$  loss.



$$\frac{1}{R_{eq}} = \frac{1}{40} + \frac{1}{40} = 0.05$$

$$R_{eq} = 20\ \Omega$$

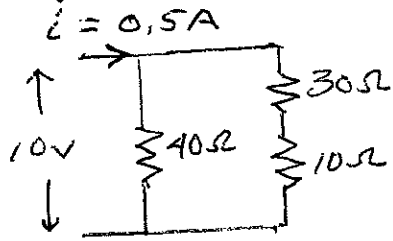


$$R_{eq} = 20 + 20 = 40\ \Omega$$

$$i = \frac{20V}{40\ \Omega} = 0.5A$$

The voltage drop across the  $20\ \Omega$  resistance is  
 $(V)_{drop} = (0.5)(20) = 10V$

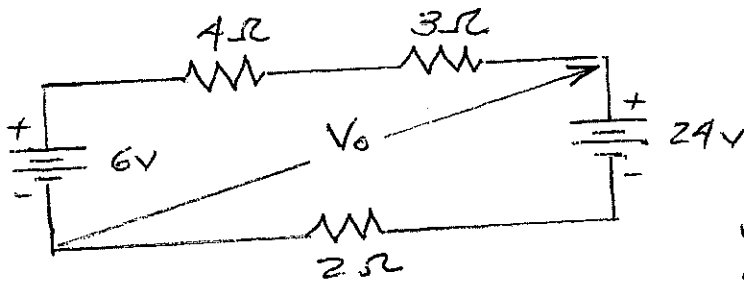
(4.14)



Since each parallel leg's resistance is the same, the current equally divides. The power loss through the 10Ω resistance is

$$i^2 R = (0.25)^2 (10) = 0.625 \text{ W}$$

4.15 Find the current and voltage for the network in Figure P4.15.



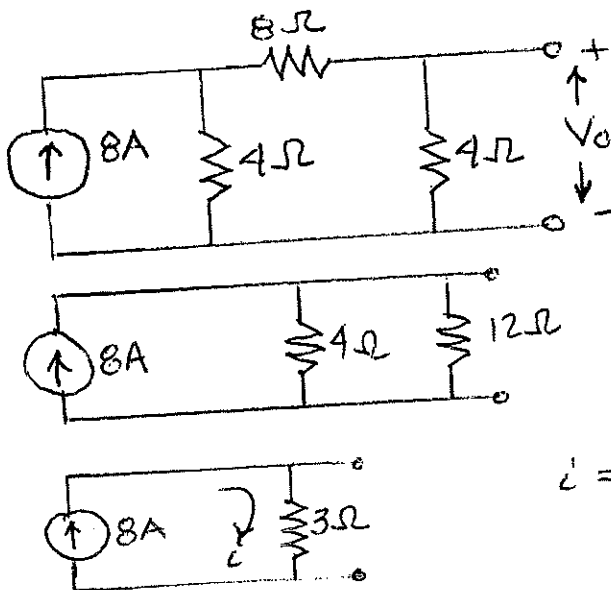
Kirchhoff's voltage law applies with voltage rises equalling voltage drops around a closed loop.

$$6 + 4i + 3i - 24 + 2i = 0$$

$$9i = 18 \quad i = 2 \text{ A}$$

$$V_0 = 24 - (2)(2) = 20 \text{ volts}$$

4.16 Determine the voltage,  $V_0$ , for the circuit in Figure P4.16.



The 8Ω + 4Ω resistance are in series  
 $R_{eq} = 12 \Omega$

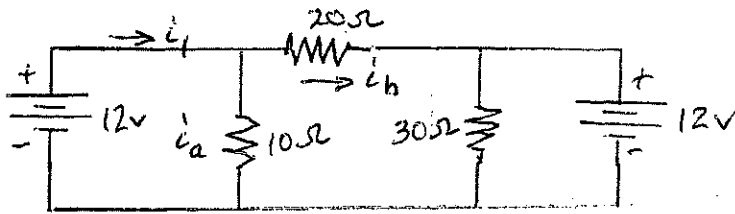
$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{12} = \frac{4}{12}$$

$$R_{eq} = 3 \Omega$$

$$i = 8 \text{ A} \quad V_0 = iR = (8)(3)$$

$$V_0 = 24 \text{ V}$$

4.17 Determine the current,  $i_1$ , for the circuit in Figure P4.17.



The  $10\Omega$  resistance is in parallel with  $12\text{V}$  supply as is the  $30\Omega$  resistance.

$$i_a = \frac{12\text{V}}{10\Omega} = 1.2\text{A}$$

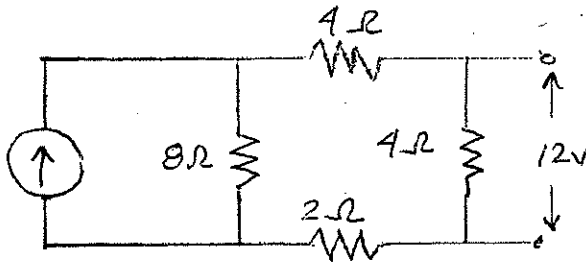
$$i_1 = i_a + i_b$$

The voltage potential across the  $20\Omega$  resistance is zero

$$\therefore i_b = \frac{0}{20} = 0$$

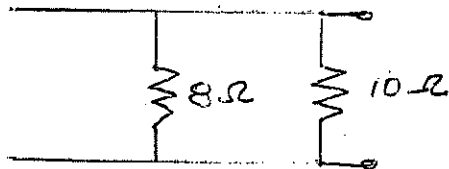
$$i_1 = 1.2\text{A}$$

4.18 Determine the value of the current source in Figure P4.18.



Find  $R_{eq}$  for the series circuit.

$$R_{eq} = 4 + 4 + 2 = 10\Omega$$



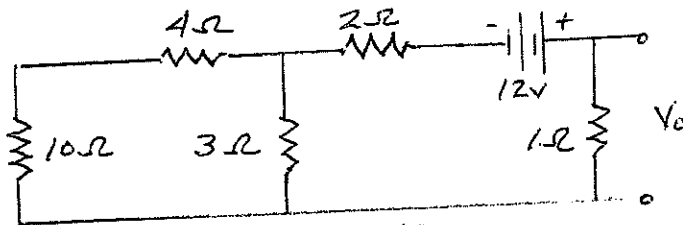
$$i_{10\Omega} = \frac{12}{10} = 1.2\text{A}$$

$$i_{8\Omega} = \frac{12}{8} = 1.5\text{A}$$

$$i_{total} = 1.2 + 1.5 = 2.7\text{A}$$

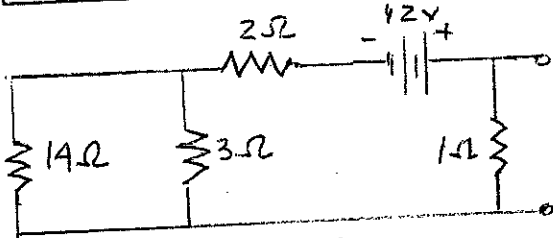
$$i_{source} = 2.7\text{A}$$

4.19 Find the voltage,  $V_o$ , for the circuit network in Figure P4.19.



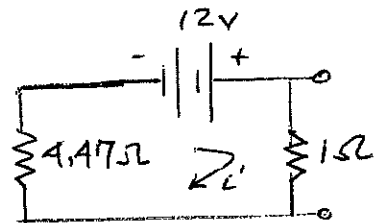
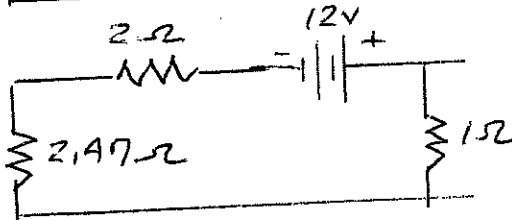
Combine series resistances

$$R_{eq} = 4 + 10 = 14\Omega$$



$$\frac{1}{R_{eq}} = \frac{1}{14} + \frac{1}{3} = 0.4048$$

$$R_{eq} = 2.47\Omega$$



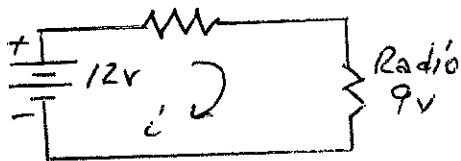
$$i = \frac{12V}{5.47\Omega} = 2.193A$$

$$V_{drop} = (2.193A)(1\Omega) = 2.193V$$

$$V \uparrow = V \downarrow \quad 12 - 2.193 = 9.807 = 0$$

$$V_o = 12 - 2.193 = 9.807V$$

4.20 Your car radio is broken, so you are using a 9-V transistor radio that uses 30 mA of current. Being an engineering student, you wish to conserve the radio's battery and want to run the radio from the car's 12-V battery. A resistor must be placed in series with the radio to reduce the car's voltage to that of the radio; what is its value? What power does the transistor radio dissipate?



$$i = 30mA = 0.030A$$

Kirchhoff's voltage law

$$12V = iR + 9V$$

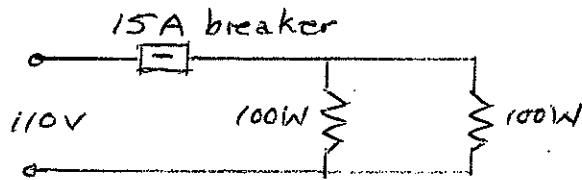
$$iR = 3 \quad R = \frac{3}{0.03} = 100\Omega$$

Power dissipated is

$$P = i^2 R = (0.03)^2 (100)$$

$$P = 0.09W$$

4.21 Many home lighting circuits have a 15-A circuit breakers with a power supply of 110 V. How many 100-W light bulbs may be placed in parallel in the circuit before the breaker trips?



Find The current flow  
Through one light

$$P = v i$$

$$100 = (110)(i) \quad i = 0.909 \text{ A}$$

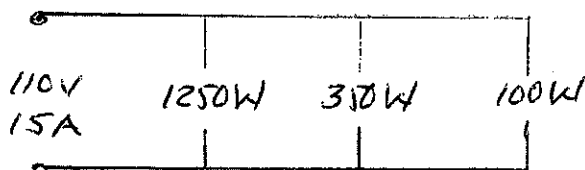
$$i_{\text{total}} = 15 = N i_{\text{bulb}}$$

$$N = \frac{15}{0.909} = 16.5 \text{ bulbs}$$

or  $N = 16$  bulbs.

Note: most actual circuit breakers are set to trip at 80% of capacity for steady-state current.

4.22 You are using a 1250-W hairdryer on a 15-A, 110-V circuit. Your younger sister comes into the room and turns on a 350-W stereo and a 100-W light, also in parallel on the same circuit. Does the circuit breaker trip?



The total power available to this circuit is

$$P = v i = (110)(15) = 1650 \text{ W}$$

The power required is  $P = 1250 + 350 + 100 = 1700 \text{ W}$   
This demands too much current and the breaker trips.

4.23 In the figure for problem 4.12 the voltage supply is 50 V, determine the current flow through the  $20 \Omega$  resistor.

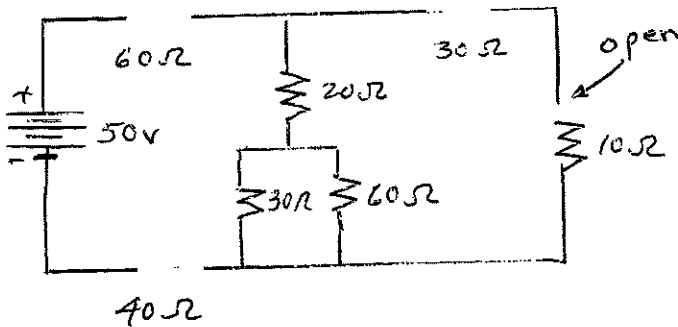
Refer to the sketch in 4.12.  $R_{\text{eq}} = 120 \Omega$

$$i_{\text{total}} = \frac{50}{120} = 0.4167 \text{ A}$$

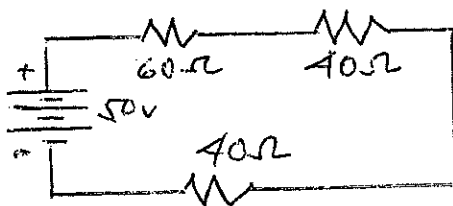
This current splits, half going through the  $20 \Omega$  resistor, half through the other branch.

$$i_{20\Omega} = 0.2083 \text{ A}$$

4.24 In the figure for problem 4.12, the voltage supply is 50 V. An open occurs across the 10  $\Omega$  resistor, find the current flow through the 20  $\Omega$  resistor.



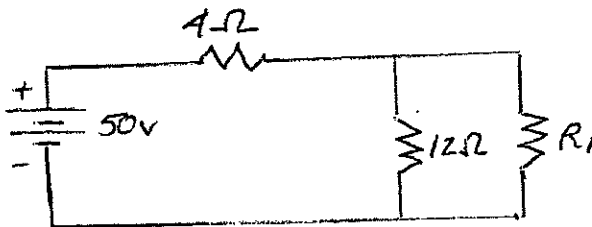
From Problem 4.12 The  $R_{eq}$  of The parallel circuit is 20  $\Omega$ . Thus, There is a series circuit



$$R_{eq} = 60 + 40 + 40 = 140 \Omega$$

$$i = \frac{V}{R_{eq}} = \frac{50}{140} = 0.357 \text{ A}$$

4.25 The power produced by the 50 V source in Figure P4.25 is 300 W. Determine  $R_1$ .



$$P = vi$$

$$300 = (50)(i)$$

$$i = 6 \text{ A}$$

$$V = i R_{eq} \quad R_{eq} = \frac{50}{6} = 8.333 \Omega$$

$$R_{eq} = 4 + R_{par} \quad \therefore R_{par} = 4.333 \Omega$$

$$\frac{1}{4.333} = \frac{1}{12} + \frac{1}{R_1}$$

$$0.2308 = 0.0833 + \frac{1}{R_1}$$

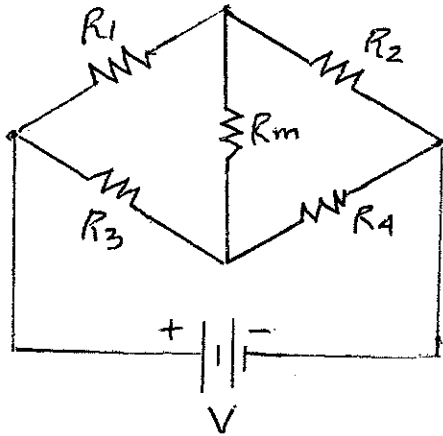
$$0.1247 = \frac{1}{R_1}$$

$$R_1 = 8.02 \Omega$$

4.26 What is meant by "balance" when referring to a bridge? How is it accomplished?

Balance means there is no current flow through the arm. It is accomplished by varying the resistance in one or more branches.

4.27 A Wheatstone bridge, as illustrated in Figure 4.10, is used to determine the value of  $R_1$ . The bridge is balanced, and  $R_3$  reads  $137.5 \Omega$ .  $R_2$  and  $R_4$  are interchanged, the bridge is balanced, and  $R_3$  now reads  $167.9 \Omega$ . Determine  $R_1$ .



$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

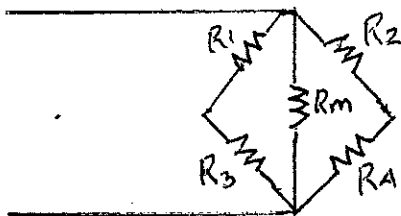
$$\frac{R_1}{137.5} = \frac{R_2}{R_4} ; \frac{R_1}{167.5} = \frac{R_4}{R_2}$$

$$\frac{167.5}{R_1} = \frac{R_1}{137.5}$$

$$R_1^2 = 23031$$

$$R_1 = 151.8 \Omega$$

4.28 In a Wheatstone bridge, the resistances  $R_3$  and  $R_4$  are equal. An unknown resistance  $R_1$  is connected to the bridge and the bridge balanced, yielding a value for  $R_2$  of 400 ohms. What is  $R_1$ 's resistance?



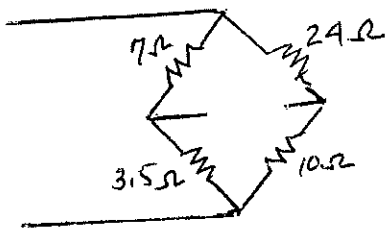
$$R_3 = R_4$$

$$R_2 = 400 \Omega$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\therefore R_1 = 400 \Omega$$

4.29 Are the following bridges (P4.29a and P4.29b) balanced?

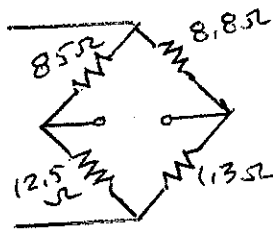


For a balanced bridge

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{7}{24} = 0.29 \quad \frac{3.5}{10} = 0.35$$

Does not balance

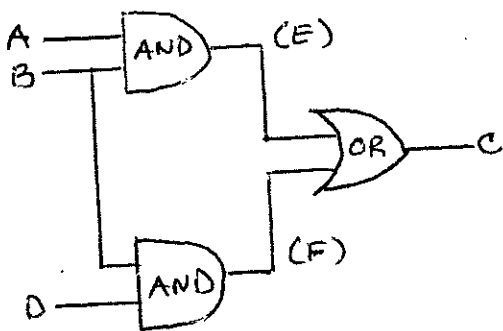


$$\frac{R_1}{R_2} = \frac{85}{8.8} = 9.66$$

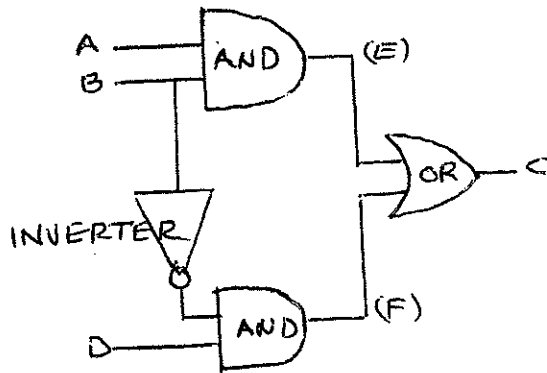
$$\frac{R_3}{R_4} = \frac{12.5}{11.3} = 9.62$$

Essentially balances.

4.30 Prepare a truth table for each of the following circuits.



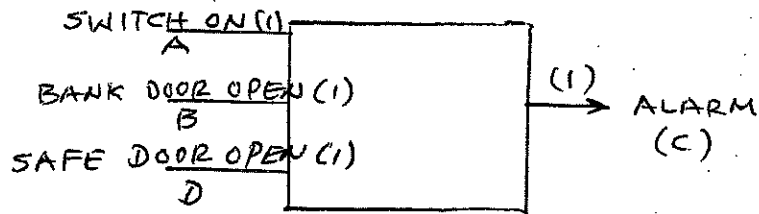
A	B	C	D	E	F
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	1	0	1	0
0	0	0	1	0	0
0	1	1	1	0	1
1	0	0	1	0	0
1	1	1	1	1	1



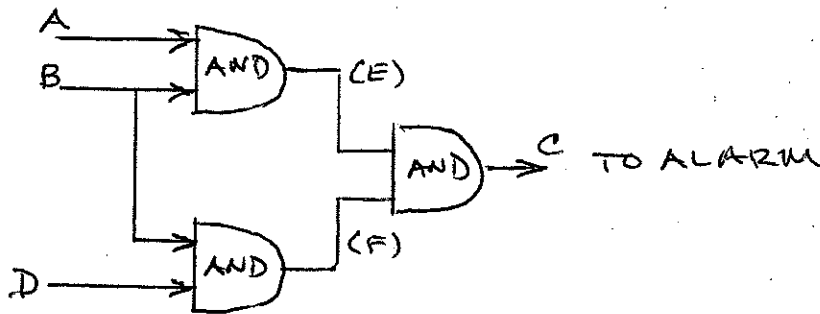
A	B	C	D	E	F
0	0	0	0	0	0
0	0	1	1	0	1
0	1	0	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	0	1	1	0	1
1	1	1	0	0	0
1	1	1	1	1	0



4.31 You have been assigned the task of creating the logic circuit for a bank alarm. The alarm is to sound, logic 1 state, if the master switch is on, if the bank door is open, and if the safe door is open.

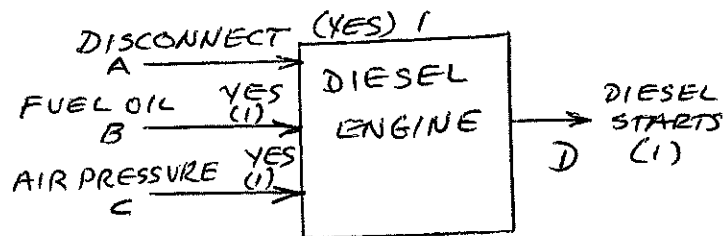


WANT SWITCH ON AND BANK DOOR OPEN AND SAFE DOOR OPEN → LOGIC 1 TO ALARM

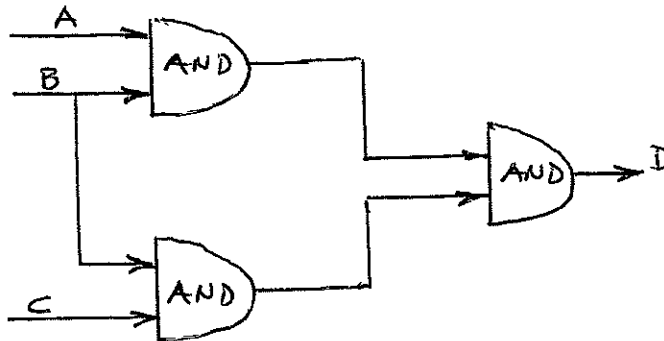


A	B	C	D	E	F
0	0	0	0	0	0
0	0	0	1	0	0
0	1	0	0	0	0
0	1	0	1	0	1
1	0	0	0	0	0
1	0	0	1	0	0
1	1	0	0	1	0
1	1	1	1	1	1

4.32 Design the logic circuit for starting an emergency diesel generator. For the generator to start, the generator must be disconnected from the bus, there must be sufficient starting air pressure, and the fuel tank must indicate there is oil. When these conditions are met, then the diesel may be started. Presume there are sensors to determine these values.

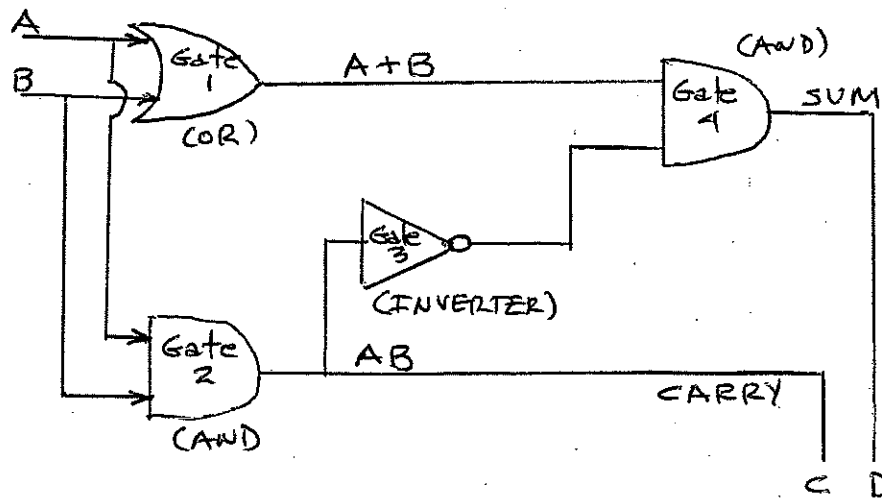


FOR THE ENGINE TO START  
 DISCONNECT AND FUEL OIL AND AIR PRESSURE  
 PRESSURE  
 MUST BE MET SIMULTANEOUSLY



A	B	C	D
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

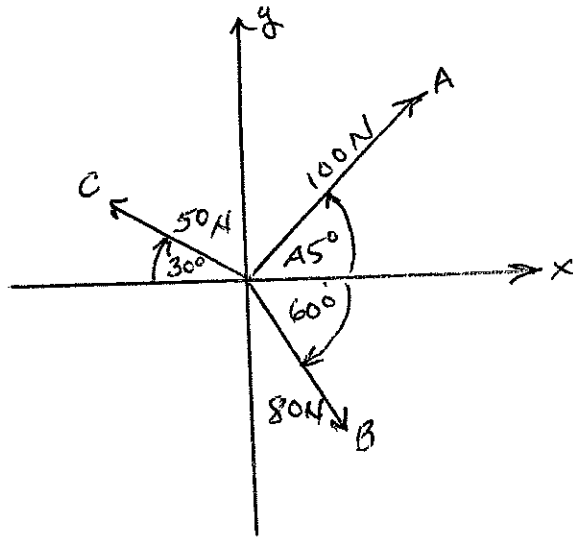
4.33 The figure below illustrates a schematic diagram for a half adder; it will add any two binary digits and produce the resulting summation as outputs. Show that it will correctly add all combinations of binary input.



Inputs		Outputs	
A	B	C	D
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

THIS PRODUCES A SUM OUTPUT (D) AND A CARRY OUTPUT (C) FROM TWO SINGLE DIGIT BINARY INPUTS, A AND B, NOTE THIS IS THE BINARY TRUTH TABLE FOR ADDITION SHOWN IN THE APPENDIX.

4.34 In Figure 4.21a, let force A = 100 N, B = 80 N and C = 50 N. Determine the resultant force's magnitude and direction.



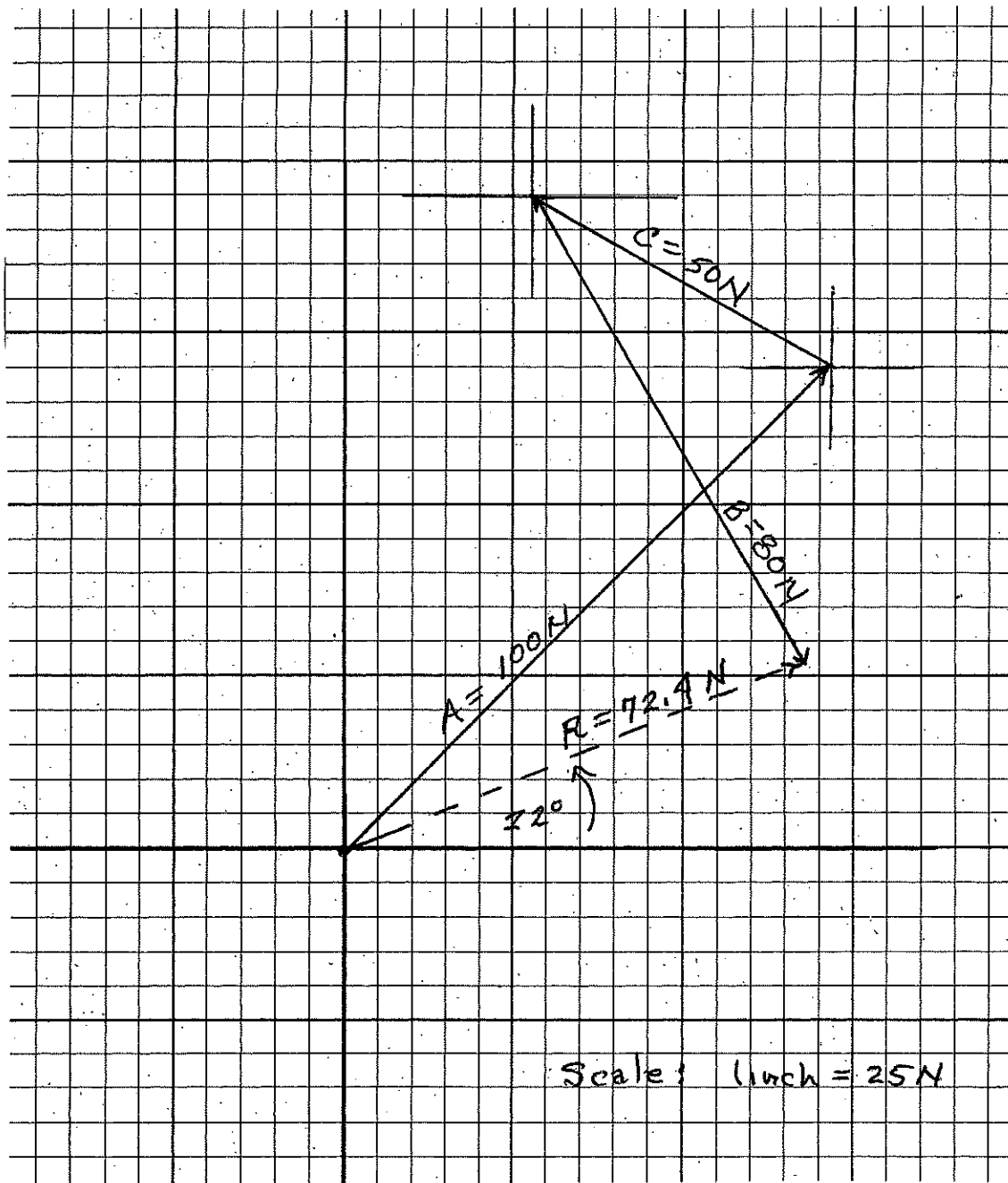
F	$F_x$	$F_y$
100	$100 \cos 45^\circ = +70,7$	$100 \sin 45^\circ = +70,7$
80	$80 \cos 60 = 40,0$	$-80 \sin 60 = -69,3$
50	$-50 \cos 30 = -43,3$	$50 \sin 30 = 25$
	$\Sigma F_x = 67,4$	$\Sigma F_y = 26,4$

$$R^2 = 67,4^2 + 26,4^2$$

$$R = 72,4 \text{ N}$$

$$\Theta = \tan^{-1} \left( \frac{26,4}{67,4} \right) = 21,4^\circ$$

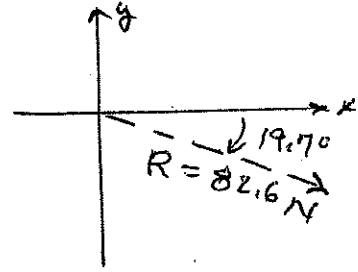
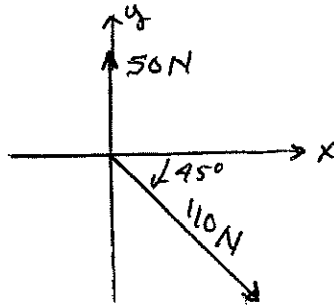
4.35. It is possible to resolve concurrent forces graphically. Perform the graphical solution for the situation in Problem 34.



Note: The graphical method requires a large drawing and careful drawing. Slight inaccuracies in any vector significantly effect the final result.

4.36. Determine the resultant force (magnitude and direction) for the following concurrent force systems:

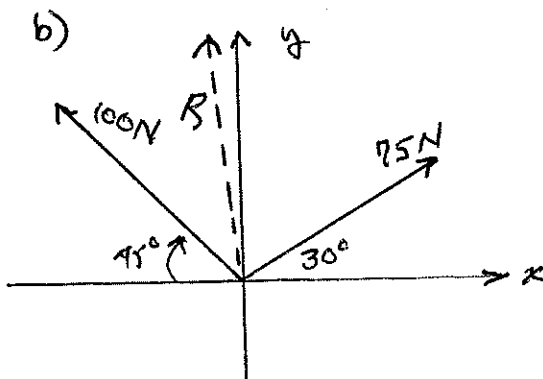
a)



$F$	$F_x$	$F_y$
50	0	+50
110	$110 \cos 45 = +77.8$	$-110 \sin 45 = -77.8$
	$\Sigma F_x = 77.8 \text{ N}$	$\Sigma F_y = -27.8 \text{ N}$

$$R^2 = 77.8^2 + 27.8^2 \quad R = 82.6 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{27.8}{77.8}\right) = 19.7^\circ \text{ in 4th quadrant or } 340.3^\circ$$



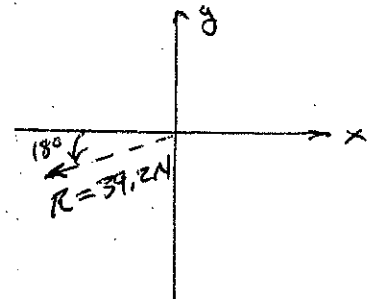
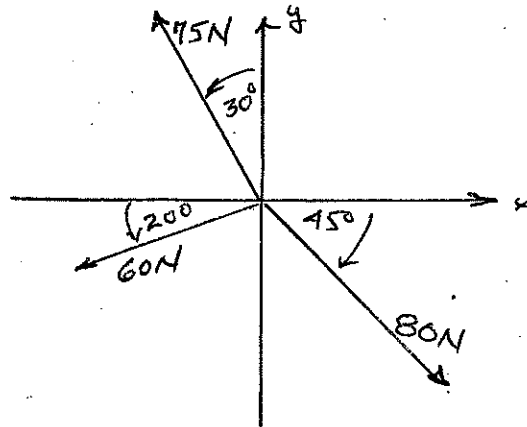
$F$	$F_x$	$F_y$
75	$75 \cos 30 = 65.0 \text{ N}$	$75 \sin 30 = +37.5 \text{ N}$
100	$-100 \cos 45 = -70.7$	$100 \sin 45 = +70.7$
	$\Sigma F_x = -5.7 \text{ N}$	$\Sigma F_y = 108.2 \text{ N}$

$$R^2 = 5.7^2 + 108.2^2$$

$$R = 108.4 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{108.2}{5.7}\right) = 87^\circ \text{ in 3rd quad. or } 93^\circ$$

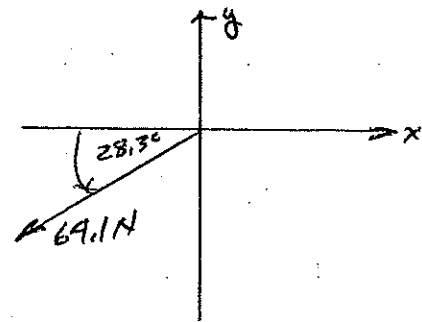
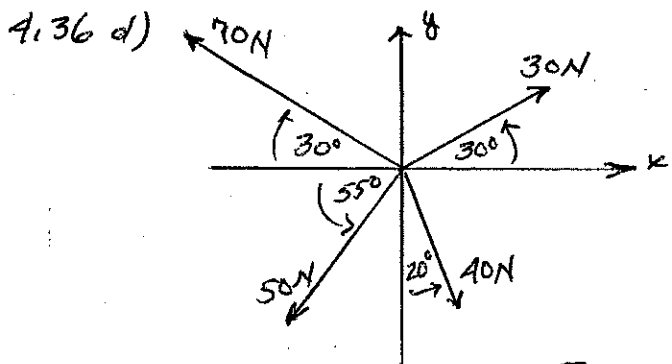
4.36 c)



F	$F_x$	$F_y$
75	$-75 \cos 60 = -37.5$	$75 \sin 60 = 65.0$
60	$-60 \cos 20 = -56.4$	$-60 \sin 20 = -20.5$
80	$80 \cos 45 = 56.6$	$-80 \sin 45 = -56.6$
	$\Sigma F_x = -37.3 \text{ N}$	$\Sigma F_y = -12.1$

$$R^2 = 37.3^2 + 12.1^2 \quad R = 39.2 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{12.1}{37.3}\right) \quad \theta = 180^\circ \text{ in 3rd quadrant or } 198^\circ$$

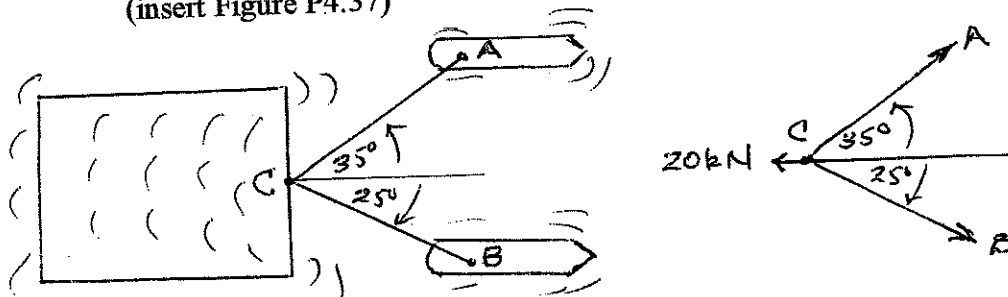


F	$F_x$	$F_y$
30	$30 \cos 30 = 26$	$30 \sin 30 = 15$
40	$40 \cos 80 = 6.9$	$-40 \sin 80 = -39.4$
50	$-50 \cos 55 = -28.7$	$-50 \sin 55 = -41.0$
70	$-70 \cos 30 = -60.6$	$70 \sin 30 = 35.0$
	$\Sigma F_x = -56.4$	$\Sigma F_y = -30.4$

$$R^2 = (56.4)^2 + (30.4)^2 \quad R = 69.1 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{30.4}{56.4}\right) \quad \theta = 28.3^\circ \text{ in 3rd quadrant}$$

- 4.37. Two tugboats are pulling a barge as illustrated below. The horizontal force acting at C is 20 000 N. Determine the force (tension) in the ropes (lines in nautical jargon) AC and BC. (insert Figure P4.37)



Perform force balances in each direction

$$\sum F_x = 0 \quad 20\,000 = CA \cos 35^\circ + CB \cos 25^\circ$$

$$20\,000 = 0.819 CA + 0.906 CB \quad (a)$$

$$\sum F_y = 0 \quad CA \sin 35^\circ = CB \sin 25^\circ$$

$$0.574 CA = 0.423 CB \quad (b)$$

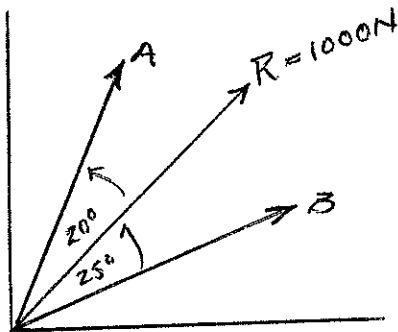
$$CB = 1.357 CA$$

$$20\,000 = 0.819 CA + (0.906)(1.357 CA)$$

$$CA = 9764 \text{ N}$$

$$CB = 13249 \text{ N}$$

- 4.38. Given that the resultant force acting on the tent peg shown in Figure 4. 22a is 1000 N and the angles are as indicated, determine the values of A and B.



$$\theta = 45^\circ \quad \alpha = 25^\circ$$

From

$$A = \frac{R \sin \alpha}{\sin(\pi - \theta)} = \frac{1000 \sin 25^\circ}{\sin(135^\circ)}$$

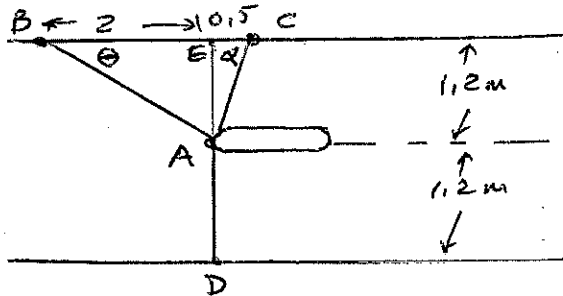
$$A = 598 \text{ N}$$

$$B = \frac{R \sin(\theta - \alpha)}{\sin(\pi - \theta)} = \frac{1000 \sin 20^\circ}{\sin(135^\circ)}$$

$$B = 484 \text{ N}$$



- 4.39. It is desired to determine the drag force on a boat hull. A model of the hull is placed in a water channel and water flows past it modeling a given hull speed. There are lines to prevent the boat from leaving the centerline of channel as well as a line to pull the boat with scales to measure the force (tension) in the lines. The readings indicate a tension of 120 N in line AB and 180 N in line AD. Determine the drag force on the hull and the tension in line AC.



$$AB = 120 \text{ N}$$

$$AD = 180 \text{ N}$$

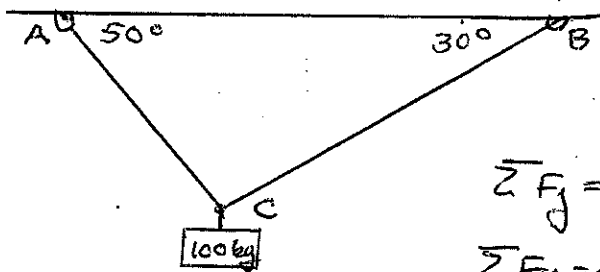
$$\theta = \tan^{-1}\left(\frac{1.2}{2.0}\right) = 31^\circ$$

$$\alpha = \tan^{-1}\left(\frac{1.2}{0.15}\right) = 67.4^\circ$$

$$\begin{aligned} \sum F_y = 0 \quad AB \sin \theta + AC \sin \alpha &= 180 \\ 120 \sin 31 + AC \sin 67.4 &= 180 \\ AC &= 128 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 \quad AB \cos \theta &= AC \cos \alpha + F_{\text{drag}} \\ 120 \cos 31 &= 128 \cos 67.4 + F_{\text{drag}} \\ F_{\text{drag}} &= 53.67 \text{ N} \end{aligned}$$

- 4.40. Two connected cables support a load as shown. Determine the tension in AC and BC.



$$\begin{aligned} F &= mg = (100 \text{ kg})(9.8 \text{ m/s}^2) \\ F &= 980 \text{ N} \end{aligned}$$

$$\sum F_y = 0 \quad 980 = AC \sin 50 + BC \sin 30$$

$$\sum F_x = 0 \quad AC \cos 50 = BC \cos 30$$

$$AC = 1.347 BC$$

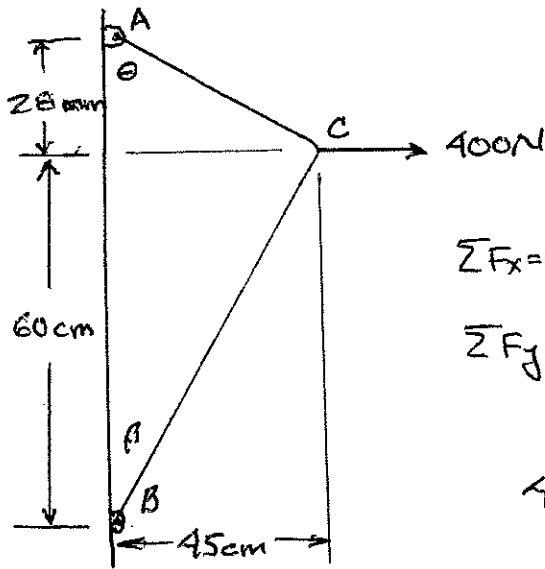
$$980 = (1.347 BC)(0.766) + (0.5) BC$$

$$980 = 1.532 BC$$

$$BC = 639.7 \text{ N}$$

$$AC = 861.6 \text{ N}$$

4.41. Two connected cables support a load as shown. Determine the tension in AC and BC.



$$\theta = \tan^{-1}\left(\frac{45}{28}\right) = 58.1^\circ$$

$$\beta = \tan^{-1}\left(\frac{45}{60}\right) = 36.9^\circ$$

$$\sum F_x = 0 \quad 400 = AC \sin 58.1^\circ + BC \sin 36.9^\circ$$

$$\sum F_y = 0 \quad AC \cos 58.1^\circ = BC \cos 36.9^\circ$$

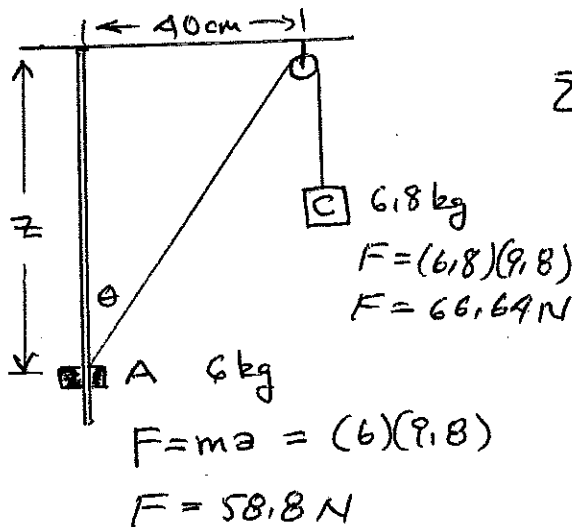
$$AC = 1.514 BC$$

$$400 = (1.514 BC)(0.849) + 0.6 BC$$

$$BC = 212.2 \text{ N}$$

$$AC = 321.3 \text{ N}$$

4.42. The 6 kilogram collar, A, may slide on the frictionless vertical rod. It is connected via a pulley to a 6.8 kilogram counterweight, C. Determine the value of height, z, for which the system is in equilibrium.



$$\sum F_y = 0$$

$$66.64 \cos \theta = 58.8$$

$$\theta = 28^\circ$$

$$\tan 28 = \frac{40}{z}$$

$$z = 75.2 \text{ cm}$$

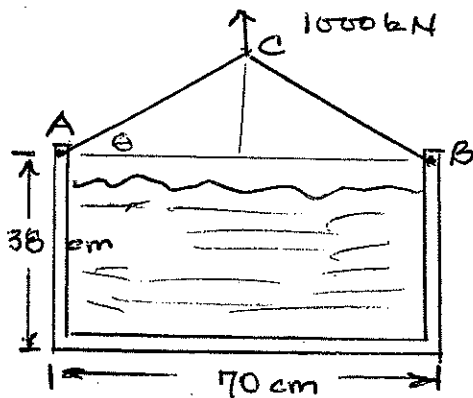
$$F = (6.8)(9.8)$$

$$F = 66.64 \text{ N}$$

$$F = ma = (6)(9.8)$$

$$F = 58.8 \text{ N}$$

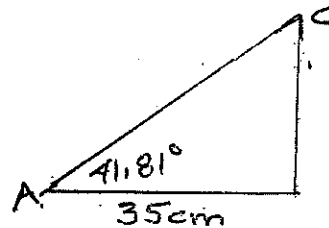
- 4.43. A container and its contents weigh 1000 kN. Determine the shortest possible sling ACB which may be used to lift the loaded container if the tension in the sling cannot exceed 750 kN.



$$AC = BC = 750 \text{ kN}$$

$$\sin \theta = \frac{500}{750}$$

$$\theta = 41.81^\circ$$

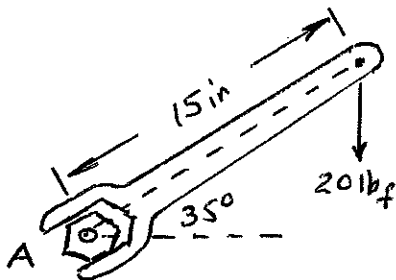


$$\cos(41.81) = \frac{35}{AC}$$

$$AC = 46.96 \text{ cm}$$

$$\therefore ACB = 93.9 \text{ cm}$$

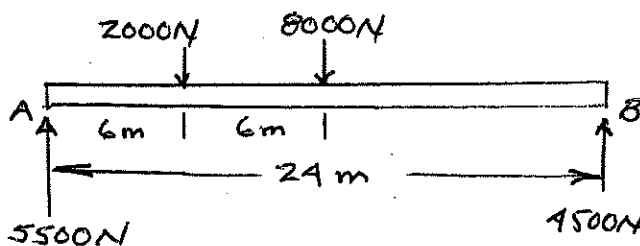
- 4.44. Determine the moment (torque) in foot-pounds about point A for the wrench.



$$M_A = (20 \text{ lbf}) \left( \frac{15}{12} \text{ ft} \right) \cos 35^\circ$$

$$M_A = 20.5 \text{ ft-lbf}$$

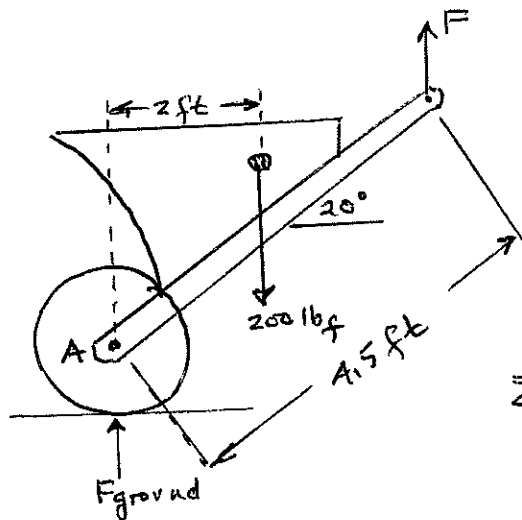
- 4.45. Determine the net moments about A and B for the beam.



$$\sum M_A = -(6 \text{ m})(2000 \text{ N}) - (8000 \text{ N})(12 \text{ m}) + (4500 \text{ N})(24 \text{ m}) = 0$$

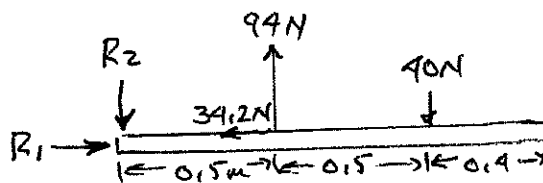
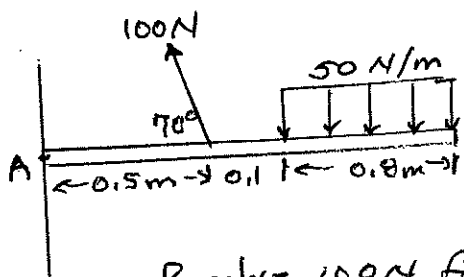
$$\sum M_B = -(24 \text{ m})(5500 \text{ N}) + (2000 \text{ N})(18 \text{ m}) + (8000 \text{ N})(12 \text{ m}) = 0$$

- 4.46. The wheelbarrow must be supported at an angle of  $20^\circ$  to the horizontal. Determine the force required to do so.



$$\begin{aligned} \sum M_A = 0 &= -(200 \text{ lbf})(2 \text{ ft}) \\ &+ (F \text{ lbf})(4.5 \cos 20^\circ \text{ ft}) \\ 400 &= 4.22 F \\ F &= 94.6 \text{ lbf} \end{aligned}$$

- 4.47. Determine the moment about A for the following sketch.



Resolve 100 N force into components

$$F_y = 100 \sin 70 = 94 \text{ N} \quad F_x = 100 \cos 70 = 34.2 \text{ N}$$

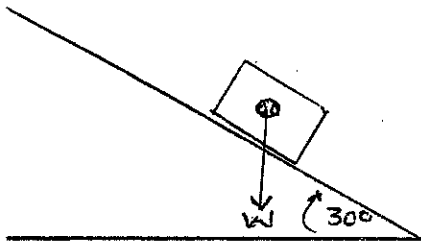
The total force at center of gravity is  $(50 \text{ N/m})(0.8 \text{ m}) = 40 \text{ N}$

$$\sum F_x = 0 \quad R_1 = 34.2 \text{ N}$$

$$\sum M_A = + (94 \text{ N})(0.5 \text{ m}) - (40 \text{ N})(1.1 \text{ m}) = 7 \text{ Nm}$$

A moment exists due to twisting action.

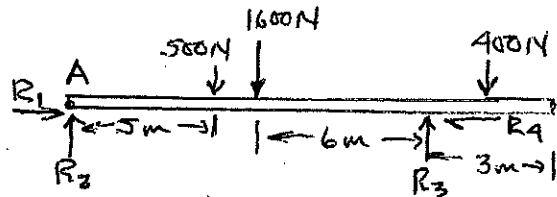
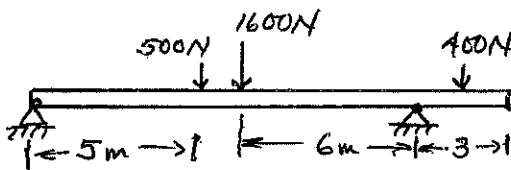
- 4.48. Find the components of the container's weight parallel and perpendicular to the inclined plane.



$$W_{\text{parallel}} = W \sin 30 = 0.5W$$

$$W_{\perp} = W \cos 30 = 0.866W$$

- 4.49. A horizontal beam is 15 m long and weighs 2000 N. It has pinned supports at the extreme left end and 3 m from the right end. In addition, there is a concentrated load of 500 N a distance of 5 m from the left end. Determine the reactions at the supports.



Assume weight is evenly distributed, 1600 N between supports and 400 N overhanging.

$$\sum F_x = 0 \quad R_1 = R_4 = 0$$

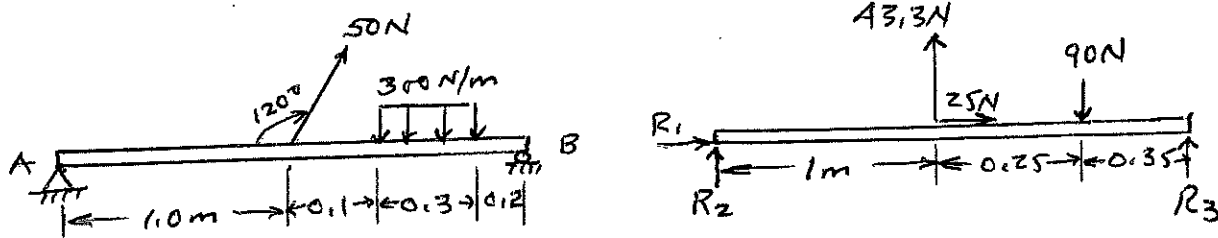
$$+\uparrow \sum F_y = 0 \quad R_2 + R_3 = 500 + 1600 + 400 = 2500 \text{ N}$$

$$\sum M_A = 0 = - (500 \text{ N})(5 \text{ m}) - (1600 \text{ N})(6 \text{ m}) + R_3(12 \text{ m}) - (400 \text{ N})(3 \text{ m})$$

$$R_3 = 1458.3 \text{ N } \uparrow$$

$$R_2 = 1041.7 \text{ N } \uparrow$$

4. 50. Refer to Figure 4.29a in Example problem 4. 5. Let the angle be 120, determine the reactions at A and B.



Resolve 50 N force into its components

$$F_x = 50 \cos 60 = 25 \text{ N} \quad F_y = 50 \sin 60 = 43.3 \text{ N}$$

The distributed load acts at the center of gravity

$$(0.3 \text{ m}) \times (300 \text{ N/m}) = 90 \text{ N}$$

$$\sum F_x = 0 = R_1 + 25 \quad \therefore R_1 = -25 \text{ N} \leftarrow \text{acting to left}$$

$$\sum F_y = 0 = R_2 + R_3 + 43.3 - 90$$

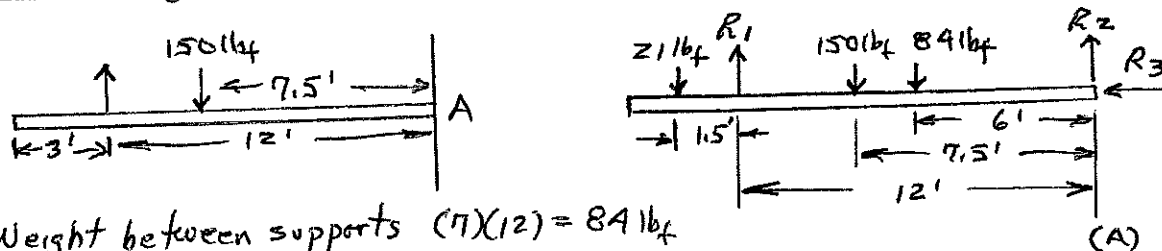
$$R_2 + R_3 = 46.7 \text{ N}$$

$$\sum M_A = 0 = (43.3 \text{ N})(1.0 \text{ m}) - (90 \text{ N})(1.25 \text{ m}) + R_3(1.6 \text{ m})$$

$$R_3 = 43.25 \text{ N} \uparrow$$

$$R_2 = 3.45 \text{ N} \uparrow$$

4. 51. A 15 foot plank that weighs 7 pounds/foot is horizontal, attached rigidly to the wall at the right end and supported by a vertical cable 3 feet from the left end. A person weighing 150 pounds is standing in the center of the plank. Determine the reactions at both end supports.



Weight between supports  $(7)(12) = 84 \text{ lb}$   
 weight of overhang  $(7)(3) = 21 \text{ lb}$

$$\sum F_x = 0$$

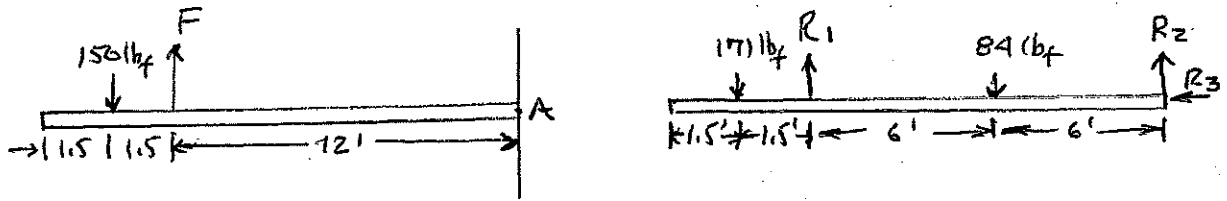
$$\sum F_y = 0 = R_1 + R_2 - 21 - 150 - 84; \quad R_1 + R_2 = 255 \text{ lb}$$

$$\sum M_A = 0 = +(84)(6) + (150)(7.5) - (R_1)(12) + (21)(13.5)$$

$$R_1 = 159.4 \text{ lb}$$

$$R_2 = 95.6 \text{ lb}$$

- 4.52. The person in Problem 4.51 now moves past the cable and is standing 1.5 feet from the left-hand side. Determine the reactions at both supports.



The person (150 lbf) plus the mass of plank in overhang (21 lbf) act 1.5 ft from left. The 12 foot section, weighing 84 lbf, acts at c of g of that section.

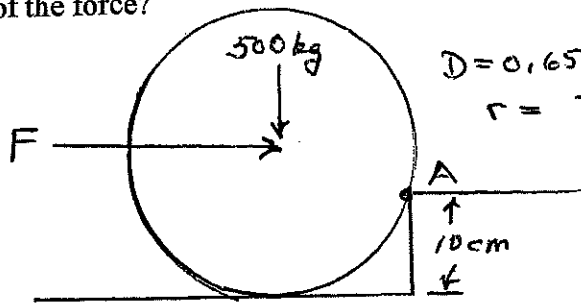
$$\sum F_x = 0 = R_3 \quad \therefore R_3 = 0$$

$$\sum F_y = 0 = R_1 + R_2 - 171 - 84; \quad R_1 + R_2 = 255$$

$$\sum M_A = 0 = + (84)(6) - (R_1)(12) + (171)(1.5)$$

$$R_1 = 234.4 \text{ lbf } \uparrow \quad R_2 = 20.6 \text{ lbf } \uparrow$$

- 4.53. A tire has a diameter of 0.65 m and supports a vertical load of 500 kg. A force, acting at the centerline, must be sufficient to cause the tire to move over a 10 cm curb. What is the amount of the force?



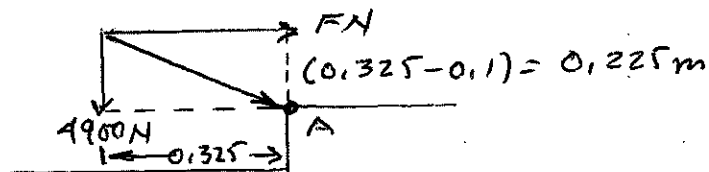
$$D = 0.65 \text{ m} = 65 \text{ cm}$$

$$r = 32.5 \text{ cm}$$

$$F = ma$$

$$F_{\text{mass}} = (500 \text{ kg})(9.8 \text{ m/s}^2)$$

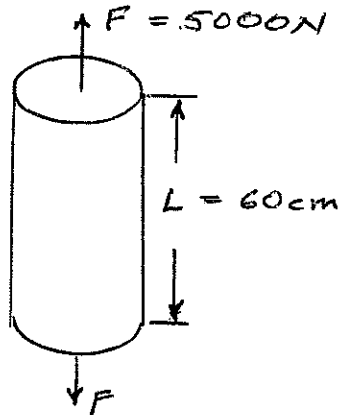
$$F = 4900 \text{ N}$$



$$\sum M_A = 0 = + (4900 \text{ N})(0.325 \text{ m}) - (F)(0.225 \text{ m})$$

$$F = 7078 \text{ N}$$

- 4.54. Referring to Figure 4.30, let the force be 5000 N and the length 60 cm and the diameter 2 cm. Calculate the normal stress and strain for steel, aluminum and brass.



$$D = 2 \text{ cm}$$

	$E$
Steel	$2.0 \times 10^8 \text{ kPa}$
Aluminum	$7.0 \times 10^7 \text{ ''}$
Brass	$9.0 \times 10^7 \text{ ''}$

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.02 \text{ m})^2}{4} = 0.00031416 \text{ m}^2$$

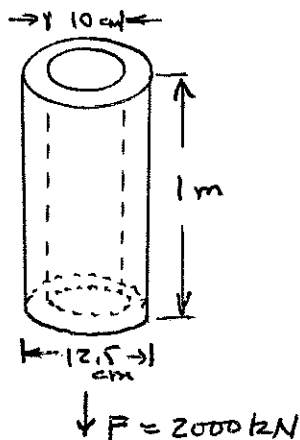
$$\sigma = \frac{F}{A} = \frac{5 \text{ kN}}{0.00031416} = 15915 \text{ kPa for all materials}$$

$$\epsilon_{\text{steel}} = \frac{\sigma}{E} = \frac{15915}{2 \times 10^8} = 8.0 \times 10^{-5}$$

$$\epsilon_{\text{Al}} = \frac{15915}{7.0 \times 10^7} = 2.3 \times 10^{-4}$$

$$\epsilon_{\text{brass}} = \frac{15915}{9 \times 10^7} = 1.8 \times 10^{-4}$$

- 4.55. Referring to Figure 4.30, let the bar be hollow with an inside diameter of 10 cm, an outside diameter of 12.5 cm and a length of 1 m. The bar is subjected to a loading of 1000 kN. Determine the elongation and the normal stress if the material is aluminum.



$$\text{Aluminum } E = 7.0 \times 10^7 \text{ kPa}$$

$$\sigma = \frac{F}{A} \quad A = \frac{\pi}{4} (0.125^2 - 0.1^2)$$

$$A = 0.005625 \text{ m}^2$$

$$\sigma = \frac{1000 \text{ kN}}{0.005625 \text{ m}^2} = 177778 \text{ kPa}$$

$$\sigma < \sigma_y \text{ so Hooke's law applies}$$

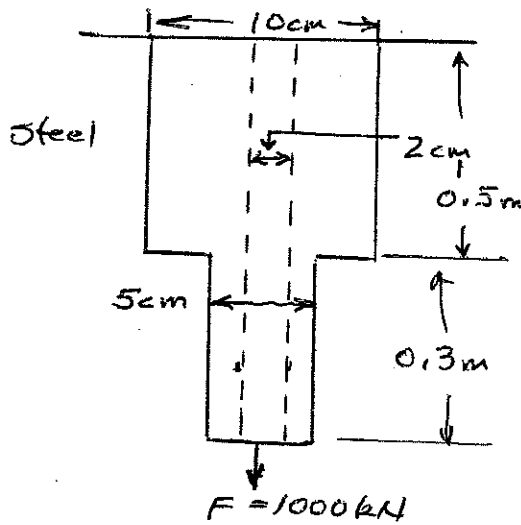
$$\epsilon = \frac{\sigma}{E} = \frac{177778 \text{ kPa}}{7 \times 10^7 \text{ kPa}}$$

$$\epsilon = 0.0025$$

$$\Delta L = \epsilon L = 0.0025 \text{ m} = 2.5 \text{ mm}$$



- 4,56. Referring to Figure 4.33, let the bar be bored to an inside diameter of 2 cm. Determine the loadings per Example 4.6.



$$E = 2 \times 10^8 \text{ kPa}$$

$$A_{0,1} = \frac{\pi}{4} (0,1^2 - 0,02^2) = 0,00754 \text{ m}^2$$

$$\sigma_{0,1} = \frac{1000 \text{ kN}}{A_{0,1}} = 132\,629 \text{ kPa}$$

$$A_{0,05} = \frac{\pi}{4} (0,05^2 - 0,02^2) = 0,00165$$

$$\sigma_{0,05} = \frac{1000}{A_{0,05}} = 606\,305 \text{ kPa}$$

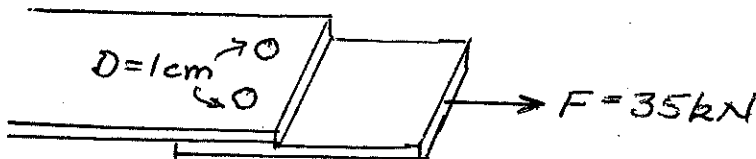
$$\epsilon_{0,1} = \frac{\sigma}{E} = \frac{132\,629}{2 \times 10^8} = 0,00066$$

$$\epsilon_{0,05} = \frac{606\,305}{2 \times 10^8} = 0,00303$$

$$\Delta L_{0,1} = (0,00066)(0,5) = 0,00033 = 0,33 \text{ mm}$$

$$\Delta L_{0,05} = (0,00303)(0,3) = 0,00091 = 0,91 \text{ mm}$$

- 4,57. Two 1-cm diameter rivets join two metal sheets together. If the force pulling the sheets is 35 kN, determine the average shear stress in each rivet.



Total cross-sectional area for both rivets, A

$$A = 2 \frac{\pi}{4} d^2 = \frac{\pi}{2} (0,01)^2 = 0,000157 \text{ m}^2$$

$$\sigma = \frac{35 \text{ kN}}{A \text{ m}^2} = 222\,817 \text{ kPa}$$

- 4.58. Two plastic parts are butted together and glued. The parts' mating surfaces have dimensions of 100 mm by 5 mm. If a tensile force of 5000 N is applied, what is the average normal stress at the interface?

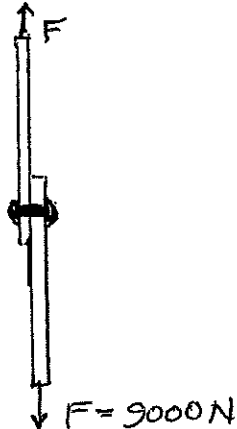


$$A = (0.1)(0.005) = 0.0005 \text{ m}^2$$

$$\sigma = \frac{F}{A} = \frac{5000 \text{ N}}{(0.0005 \text{ m}^2)}$$

$$\sigma = 10000 \text{ kPa}$$

- 4.59. A riveted connection must support a load of 9000 N in single shear. What diameter steel rivet should be used if the factor of safety is 1.5?



$$F_s = 1.5 \quad \sigma_y = 358 \text{ MPa}$$

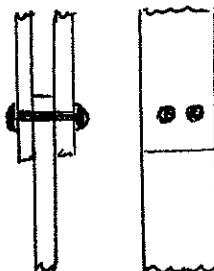
$$\sigma_{\text{allow}} = \frac{358}{1.5} = 238.67 \text{ MPa}$$

$$\sigma = \frac{F}{A} = 238670 = \frac{9 \text{ kN}}{A \text{ m}^2}$$

$$A = 3.771 \times 10^{-5} \text{ m}^2 = \frac{\pi}{4} d^2$$

$$d = 0.0069 \text{ m} = 6.9 \text{ mm}$$

- 4.60. A riveted connection with two rivets in double shear supports a 1500 kN load. What is the rivet diameter if the rivets are steel and the factor of safety is 1.5?



$$F_s = 1.5 \quad \sigma_y = 358 \text{ MPa}$$

$$\sigma_{\text{allow}} = \frac{358}{1.5} = 238.7 \text{ MPa}$$

$$\sigma = \frac{F}{A} \quad A = \frac{1500 \text{ kN}}{238700 \text{ kN/m}^2}$$

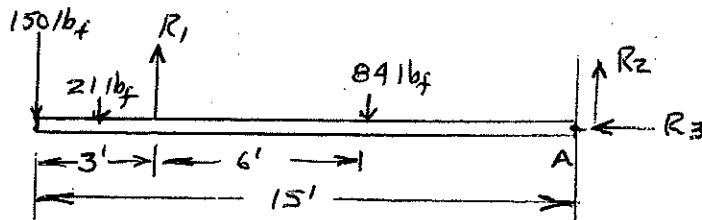
$$A = 0.006284 \text{ m}^2$$

There are two rivets in double shear

$$A = (2 \times 2) \frac{\pi}{4} d^2 = \pi d^2 = 0.006284$$

$$d = 0.0447 \text{ m} = 4.47 \text{ cm}$$

- 4.61. The cable in Problem 51 has an allowable stress of 58 500 kPa. Determine its diameter for the worst load condition.



Worst case occurs when person is at extreme left.

$$\sum F_x = 0 \quad \therefore R_3 = 0$$

$$\sum F_y = 0 = R_1 + R_2 - 150 - 21 - 84$$

$$R_1 + R_2 = 255 \text{ lb}_f$$

$$\sum M_A = 0 = + (84)(6) - (R_1)(12) + (21)(13.5) + (150)(15)$$

$$R_1 = 253 \text{ lb}_f$$

$$R_1 = (253.1 \text{ lb}_f) \left( 4.4482 \frac{\text{N}}{\text{lb}_f} \right) = 1125.8 \text{ N}$$

$$\sigma = 58500 \text{ kPa} = \frac{F}{A} = \frac{1.1258 \text{ kN}}{A \text{ m}^2}$$

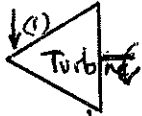
$$A = 1.9244 \times 10^{-5} \text{ m}^2 = \frac{\pi}{4} d^2$$

$$d = 0.00495 \text{ m}$$

$$d = 4.95 \text{ mm}$$

4.62 A steam turbine has an inlet steam flow of 4 kg/s with a density of 20 kg/m<sup>3</sup>. The inlet diameter is 10 cm, and the outlet diameter is 20 cm. The outlet density is 10 kg/m<sup>3</sup>. Determine the inlet and outlet velocities.

$$\dot{m}_1 = 4 \text{ kg/s} \quad \rho_1 = 20 \text{ kg/m}^3 \\ d_1 = 10 \text{ cm}$$



$$\dot{m}_2 = \dot{m}_1, \quad \rho_2 = 10 \text{ kg/m}^3 \\ d_2 = 20 \text{ cm}$$

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (0.2)^2 = 0.031416 \text{ m}^2$$

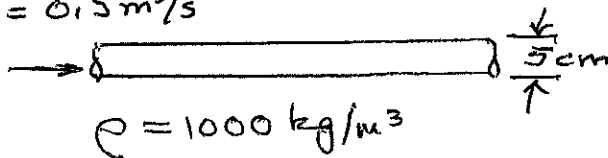
$$\dot{m} = \rho A v \\ v = \dot{m} / \rho A$$

$$v_1 = \frac{(4 \text{ kg/s})}{(20 \text{ kg/m}^3)(0.00785 \text{ m}^2)} = 25.46 \text{ m/s}$$

$$v_2 = \frac{(4)}{(10)(0.031416)} = 12.73 \text{ m/s}$$

4.63 Water with a density of 1000 kg/m<sup>3</sup> flows steadily through a pipe with an internal diameter of 5 cm. The volume flow rate is 0.5 m<sup>3</sup>/s. Determine the mass flow and velocity.

$$\dot{V} = 0.5 \text{ m}^3/\text{s}$$



$$\dot{m} = \rho A v \quad A v = \dot{V} \text{ volume flow rate}$$

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(0.5 \text{ m}^3/\text{s}) = 500 \text{ kg/s}$$

$$\dot{m} = \rho A v$$

$$(500 \text{ kg/s}) = (1000 \text{ kg/m}^3) \left( \frac{\pi}{4} (0.05)^2 \text{ m}^2 \right) (v \text{ m/s})$$

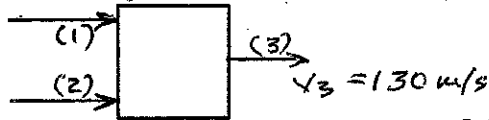
$$v = 254.6 \text{ m/s}$$

4.64 Two gaseous streams containing the same fluid enter a mixing chamber and leave as a single stream. For the first gas the entrance conditions are  $A_1 = 500 \text{ cm}^2$ ,  $v_1 = 130 \text{ m/s}$ ,  $\rho_1 = 1.60 \text{ kg/m}^3$ . For the second gas the entrance conditions are  $A_2 = 400 \text{ cm}^2$ ,  $\dot{m}_2 = 8.84 \text{ kg/s}$ , and  $\rho_2 = 1.992 \text{ kg/m}^3$ . The exit stream conditions is  $v_3 = 130 \text{ m/s}$  and  $\rho_3 = 2.288 \text{ kg/m}^3$ . Determine the total mass flow leaving the chamber and the velocity of the second gas entering the chamber.

$$A_1 = 500 \text{ cm}^2$$

$$v_1 = 130 \text{ m/s}$$

$$\rho_1 = 1.6 \text{ kg/m}^3$$



$$A_2 = 400 \text{ cm}^2$$

$$\dot{m}_2 = 8.84 \text{ kg/s}$$

$$\rho_2 = 1.992 \text{ kg/m}^3$$

$$v_3 = 130 \text{ m/s}$$

$$\rho_3 = 2.288 \text{ kg/m}^3$$

Conservation of Mass  
 $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$\dot{m}_1 = \rho_1 A_1 v_1 = (1.6 \text{ kg/m}^3) \left( \frac{500 \text{ cm}^2}{10000 \frac{\text{cm}^2}{\text{m}^2}} \right) (130 \text{ m/s})$$

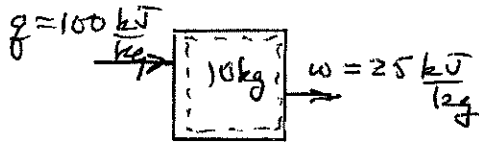
$$\dot{m}_1 = 10.4 \text{ kg/s}$$

$$\dot{m}_3 = 19.24 \text{ kg/s}$$

$$\dot{m}_2 = \rho_2 A_2 v_2 = (1.992) \left( \frac{400}{10000} \right) (v_2) = 8.84$$

$$v_2 = 110.9 \text{ m/s}$$

4.65 One hundred kJ/kg is added to 10 kg of a fluid while 25 kJ/kg of work is extracted. Determine the change of internal energy. Find the temperature change if the substance is: a) water; b) air.



The Conservation of Energy for closed systems is

$$Q = m(u_2 - u_1) + m \frac{(v_2^2 - v_1^2)}{2} + m(z_2 - z_1)g + W$$

The change in velocity is zero, The change in elevation is zero, hence  $Q = m(\Delta u) + W$ ; divide by  $m$

$$q = \Delta u + w$$

$$100 = \Delta u + 25 \quad \Delta u = 75 \text{ kJ/kg}$$

$$\Delta u = c \Delta T$$

$$\Delta T = \Delta u / c$$

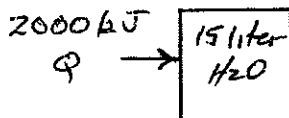
$$c_{\text{air}} = 0.7176 \text{ kJ/kg}\cdot\text{K}$$

$$\Delta T_{\text{air}} = \frac{75}{0.7176} = 104.5^\circ\text{C}$$

$$c_{\text{water}} = 4.186 \text{ "}$$

$$\Delta T_{\text{water}} = \frac{75}{4.186} = 17.9^\circ\text{C}$$

4.66 A container holds 15 liters of water. If 2000 kJ of heat are added, what is the temperature change?



$$Q = \Delta U + \Delta KE + \Delta PE + W$$

$$\Delta KE = 0, \Delta PE = 0, W = 0 \quad (V=C)$$

$$Q = \Delta U = m c \Delta T$$

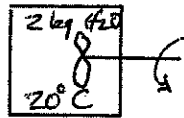
The density of water is  $1000 \text{ kg/m}^3$ ; There are  $1000 \text{ lit/m}^3$  so each liter contains  $1 \text{ kg}$  of water. Thus,  $m = 15 \text{ kg}$

$$c_{\text{water}} = 4.186 \text{ kJ/kg}\cdot\text{K}$$

$$2000 = (15)(4.186)(\Delta T)$$

$$\Delta T = 31.9^\circ\text{K} = 31.9^\circ\text{C}$$

4.67 An adiabatic tank contains 2 kg of water at 20 C and receives 20 kN . m of work from a paddle wheel. Determine the final temperature.



Closed System

$$Q = \Delta U + \Delta KE + \Delta PE + W + W_p$$

$$Q = 0 \text{ adiabatic}$$

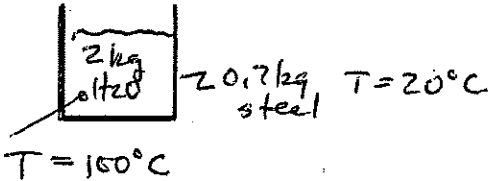
$$\Delta KE = 0, \Delta PE = 0, W = 0 \text{ (V=C)}$$

$$0 = m(u_2 - u_1) + W_p = mc(T_2 - T_1) + W_p$$

$$0 = (2 \text{ kg})(4.186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(T_2 - 293 \text{ K}) - 20$$

$$T_2 = 295.4^\circ \text{K} = 22.4^\circ \text{C}$$

4.68 Two kg of boiling water (100 C) are poured into a 0.7 kg steel container at 20 C. What will their final equilibrium temperature be, assuming no losses to the surroundings?



Conservation of Energy

$$U_{\text{steel}} + U_{\text{water}} = U_{\text{final}}$$

$$m_s c_s T_s + m_w c_w T_w = m_s c_s T_3 + m_w c_w T_3$$

$$(0.7 \text{ kg}) \left( 0.419 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \right) (293 \text{ K}) + (2) (4.186) (373) =$$

$$T_3 \left( (2)(4.186) + (0.7)(0.419) \right)$$

$$T_3 = 370.3 \text{ K} = 97.3^\circ \text{C}$$

4.69 Determine the energy release from burning: a) one metric ton of coal; b) one liter of gasoline (specific gravity = 0.836); c) 5 kg of natural gas.

$$Q_{\text{released}} = (m \text{ kg}) \left( \text{hep} \frac{\text{kJ}}{\text{kg}} \right)$$

$$a) \quad (1000 \text{ kg}) \left( 27900 \frac{\text{kJ}}{\text{kg}} \right) = 2.79 \times 10^7 \text{ kJ}$$

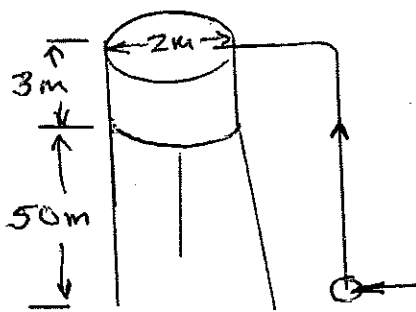
$$b) \quad 1 \text{ liter } H_2O = 1 \text{ kg} \quad \text{sp gravity} = \rho / \rho_{H_2O}$$

$$1 \text{ liter gasoline} = (1)(0.836) = 0.836 \text{ kg}$$

$$(0.836)(44800) = 37453 \text{ kJ}$$

$$c) \quad (5)(57450) = 287250 \text{ kJ}$$

4.70 A tank with a diameter of 2 m and a height of 3 m is located 50 m above the ground. It is filled with water from a pump located on the ground. What energy is required to fill the tank? If the tank fills in one hour, what average power is required?



The work is the same as lifting a full tank 50 m. The mass in the tank is

$$m = \rho V = (1000 \frac{\text{kg}}{\text{m}^3}) \left( \frac{\pi}{4} 2^2 \times 3 \right)$$

$$m = 9424.8 \text{ kg}$$

$$Q = \Delta U + \Delta KE + \Delta PE + W$$

$$Q = 0, \text{ no heat transfer, } \Delta KE = 0, \Delta U = 0$$

$$W = -\Delta PE = -mg(z_2 - z_1)$$

$$W = -(9424.8 \text{ kg})(9.8 \text{ m/s}^2)(50 - 0 \text{ m}) \div 1000 \text{ J/kJ}$$

$$W = -4618 \text{ kJ}$$

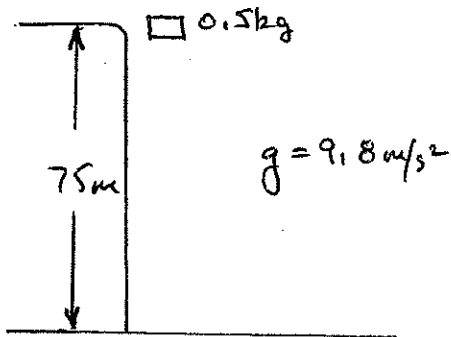
Power,  $\dot{W}$ , is

$$\dot{W} = \frac{W}{t} = \frac{-4618 \text{ kJ}}{3600 \text{ s}} = -1.28 \text{ kW}$$



4.71 A 0.5 kg container is dropped from the top of a 75-m tall building. Determine its kinetic energy and potential energy at:

- the moment it is dropped;
- after it has fallen 50 m;
- the instant before hitting the ground.



a) no initial velocity,  $KE_1 = 0$

$$PE_1 = mgz_1$$

$$PE_1 = (0.5 \text{ kg}) \times (9.8 \frac{\text{m}}{\text{s}^2}) \times (75 \text{ m})$$

$$PE_1 = 367.5 \text{ J}$$

$$b) PE_2 = (0.5) \times (9.8) \times (25)$$

$$PE_2 = 122.5 \text{ J}$$

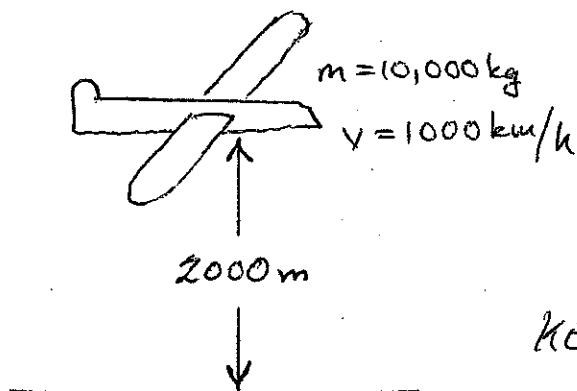
The conversion is from potential energy to kinetic energy

$$KE_2 = PE_1 - PE_2 = 245 \text{ J}$$

c) All potential energy is converted to kinetic energy

$$PE = 0 \quad KE = 367.5 \text{ J}$$

4.72 An airplane weighting 10 000 kg is flying 2000 m above the earth's surface at 1000 km/h. Determine the plane's kinetic and potential energies.



Find The velocity in m/s

$$v = (1000 \frac{\text{km}}{\text{h}}) \times (\frac{1000 \text{ m}}{\text{km}}) \times (\frac{1 \text{ h}}{3600 \text{ s}})$$

$$v = 277.8 \text{ m/s}$$

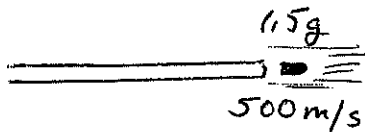
$$KE = \frac{1}{2} mv^2 = \frac{1}{2} \frac{(10000 \text{ kg}) (277.8 \text{ m/s})^2}{(1000 \text{ J/kJ})}$$

$$KE = 3.858 \times 10^5 \text{ kJ}$$

$$PE = mgz = \frac{(10000 \text{ kg}) (9.8 \text{ m/s}^2) (2000 \text{ m})}{(1000 \text{ J/kJ})}$$

$$PE = 196000 \text{ kJ}$$

4.73 A rifle bullet has a mass of 1.5 g and leaves the barrel of the gun at 500 m/s. Determine its kinetic energy.

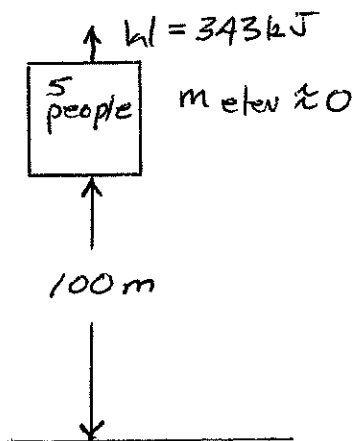


$$KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} \left( \frac{1.5 \text{ g}}{1000 \text{ g/kg}} \right) (500 \text{ m/s})^2$$

$$KE = 187.5 \text{ J}$$

4.74 Five people are lifted on an elevator a distance of 100 m. The work is found to be 343 kJ. Determine the average mass per person.



Closed system, 1<sup>st</sup> Law

$$Q = \Delta U + \Delta KE + \Delta PE + W$$

$$Q = 0, \Delta KE = 0, \Delta U = 0$$

$$-W = \Delta PE = m_{\text{total}} g \Delta z$$

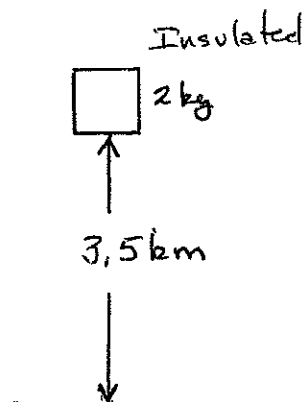
$$-(-343 \text{ kJ}) = \frac{(m_{\text{t}} \text{ kg})(9.8 \text{ m/s}^2)(100 \text{ m})}{(1000 \text{ J/kJ})}$$

$$m_{\text{t}} = 350 \text{ kg (five people)}$$

$$5m = 350$$

$$m = 70 \text{ kg per person}$$

4.75 An adiabatically insulated 2 kg container is dropped from a balloon 3.5 km above the earth. Upon impact with the ground the box remains intact; the volume remains the same, so no work is done on it. What is the change of internal energy of the box after impact?



Closed System, 1<sup>st</sup> Law

$$Q = \Delta U + \Delta KE + \Delta PE + W$$

$$Q = 0 \text{ insulated adiabatically}$$

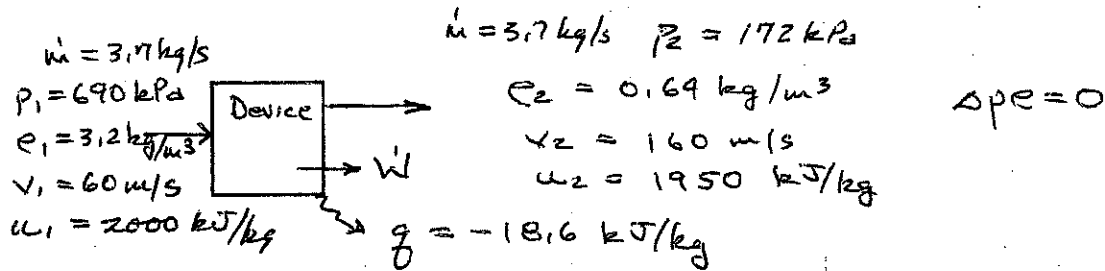
$$W = 0 \quad v = c$$

$$\Delta KE = 0 \quad \text{velocity initially } \neq \text{ on ground is zero}$$

$$\Delta U = -\Delta PE = -mg(z_2 - z_1) = +mg(z_1 - z_2)$$

$$\Delta U = \frac{(2 \text{ kg})(9.8 \text{ m/s}^2)(3500 \text{ m})}{(1000 \text{ J/kJ})} = 68.6 \text{ kJ}$$

4.76 A fluid enters a device with a steady flow of 3.7 kg/s, an initial pressure of 690 kPa, an initial density of 3.2 kg/m<sup>3</sup>, an initial velocity of 60 m/s, and initial specific internal energy of 2000 kJ/kg. It leaves at 172 kPa,  $\rho_2 = 0.64 \text{ kg/m}^3$  and  $u = 1950 \text{ kJ/kg}$ . The heat loss is found to be 18.6 kJ/kg. Find the power (work per unit time) in kilowatts.



Open System, 1st Law

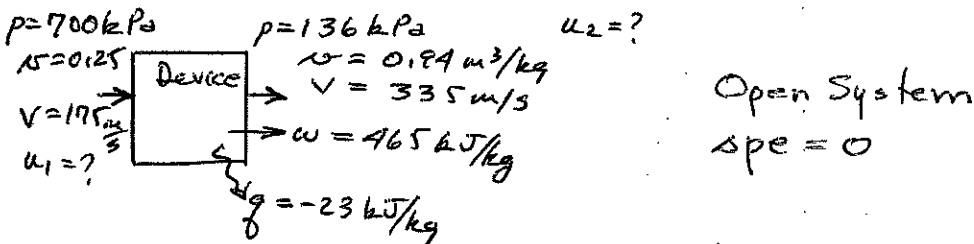
$$\dot{Q}' + \dot{m} \left( u_1 + p_1/\rho_1 + k e_1 + p e_1 \right) = \dot{W} + \dot{m} \left( u_2 + p_2/\rho_2 + k e_2 + p e_2 \right)$$

$$(3.7 \text{ kg/s}) (-18.6 \text{ kJ/kg}) + (3.7 \text{ kg/s}) \left[ 2000 + 690/3.2 + \frac{60^2}{2(1000)} \frac{\text{kJ}}{\text{kg}} \right]$$

$$= \dot{W} + (3.7) \left( 1950 + 172/0.64 + \frac{160^2}{2(1000)} \right)$$

$$\dot{W} = -121.1 \text{ kW}$$

4.77 A fluid at 700 kPa, with a specific volume of 0.25 m<sup>3</sup>/kg and a velocity of 175 m/s enters a device. Heat loss from the device by radiation is 23 kJ/kg. The work done by the fluid is 465 kJ/kg. The fluid exits at 136 kPa, 0.94 m<sup>3</sup>/kg and 335 m/s. Determine the change of internal energy.



$$\dot{Q}' + \dot{m} \left( u_1 + p_1 v_1 + k e_1 + p e_1 \right) = \dot{W} + \dot{m} \left( u_2 + p_2 v_2 + k e_2 + p e_2 \right)$$

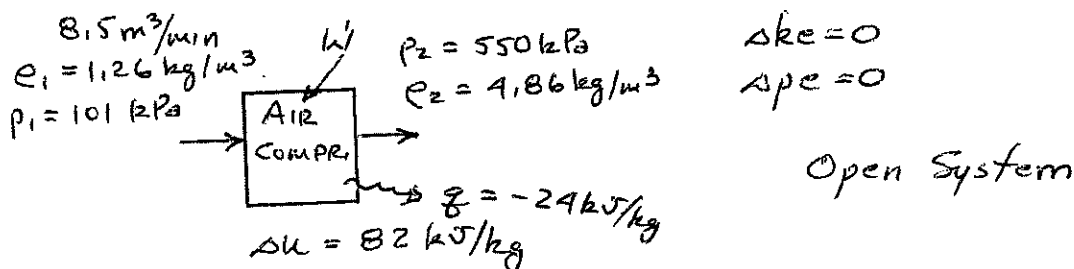
divide by  $\dot{m}$

$$q + \left( u_1 + p_1 v_1 + \frac{v_1^2}{2} \right) = w + \left( u_2 + p_2 v_2 + \frac{v_2^2}{2} \right)$$

$$-23 + (700)(0.25) + \frac{175^2}{2(1000)} = 465 + (u_2 - u_1) + (136)(0.94) + \frac{335^2}{2(1000)}$$

$$\Delta u = -481.6 \text{ kJ/kg}$$

4.78 An air compressor handles  $8.5 \text{ m}^3/\text{min}$  of air with a density of  $1.26 \text{ kg/m}^3$  and a pressure of one atmosphere and discharges it at  $550 \text{ kPa}$  with a density of  $4.86 \text{ kg/m}^3$ . The change in the specific internal energy across the compressor is  $82 \text{ kJ/kg}$ , and the heat loss by cooling is  $24 \text{ kJ/kg}$ . Neglecting changes in kinetic and potential energies, find the power in KW.



$$\dot{Q} + \dot{m} \left( u_1 + p_1/\rho_1 + \cancel{ke_1} + \cancel{pe_1} \right) = \dot{W} + \dot{m} \left( u_2 + p_2/\rho_2 + \cancel{ke_2} + \cancel{pe_2} \right)$$

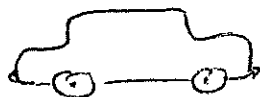
$$\dot{m} = (8.5 \text{ m}^3/\text{min}) (1.26 \text{ kg/m}^3) \left( \frac{1}{60} \frac{\text{s}}{\text{min}} \right) = 0.1785 \frac{\text{kg}}{\text{s}}$$

$$(0.1785 \frac{\text{kg}}{\text{s}}) (-24 \frac{\text{kJ}}{\text{kg}}) + (0.1785 \frac{\text{kg}}{\text{s}}) \left[ \frac{(101 \text{ kN/m}^2)}{1.26 \text{ kg/m}^3} \right] = \dot{W} +$$

$$0.1785 \frac{\text{kg}}{\text{s}} \left[ 82 \frac{\text{kJ}}{\text{kg}} + \frac{(550 \text{ kN/m}^2)}{4.86 \text{ kg/m}^3} \right]$$

$$\dot{W} = -24.8 \text{ kW}$$

4.79 Calculate the kinetic energy of a  $1200 \text{ kg}$  automobile moving at  $60 \text{ mph}$ .



$$v = 60 \text{ mph}$$

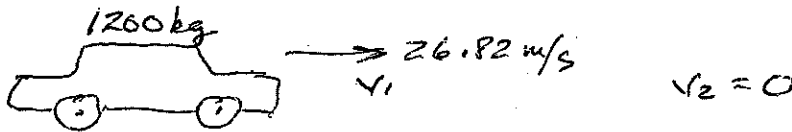
$$m = 1200 \text{ kg}$$

$$v = 60 \times 0.44703 = 26.82 \text{ m/s}$$

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} (1200 \text{ kg}) (26.82 \frac{\text{m}}{\text{s}})^2$$

$$KE = 431.6 \text{ kJ}$$

4.80 The automobile in Problem 4.79 is stopped. The brakes have an average specific heat of  $0.92 \text{ kJ/kg}\cdot\text{K}$ . Assume that one-half of the energy is adiabatically absorbed by the brakes, which have a collective mass of  $6 \text{ kg}$ . Determine the temperature rise of the brakes.



Consider brakes as the closed system

$$Q = \Delta U + \Delta KE + \Delta PE + W$$

$$W = 0, \Delta PE = 0$$

Half of the kinetic energy is absorbed by the brakes. The other half is dissipated as heat.

$$\Delta U = -\frac{1}{2} KE = -\frac{1}{2} \left[ \frac{1}{2} m(v_2^2 - v_1^2) \right]$$

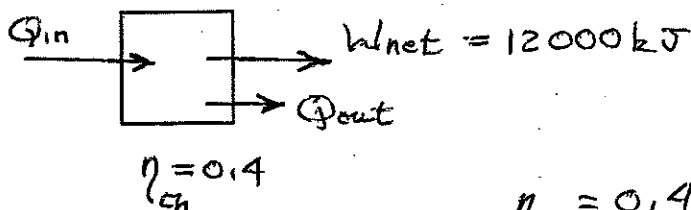
$$\Delta U = \frac{(1200 \text{ kg})(26.82 \text{ m/s})^2}{4} = 215.8 \text{ kJ}$$

$$\Delta U = 215.8 \text{ kJ} = m c \Delta T$$

$$215.8 \text{ kJ} = (6 \text{ kg})(0.92 \text{ kJ/kg}\cdot\text{K})(\Delta T \text{ } ^\circ\text{K})$$

$$\Delta T = 39.1 \text{ } ^\circ\text{K} = 39.1 \text{ } ^\circ\text{C}$$

4.81 A heat power cycle with a thermal efficiency of  $0.4$  produces  $12000 \text{ kJ}$  of net work. Determine the heat added and heat rejected per cycle.



$$\eta_{th} = 0.4 = \frac{W_{net}}{Q_{in}}$$

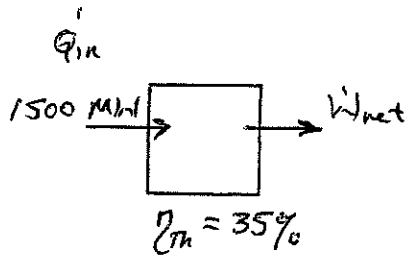
$$Q_{in} = \frac{12000}{0.4} = 30000 \text{ kJ}$$

$$Q_{in} + Q_{out} = W_{net}$$

$$30000 + Q_{out} = 12000$$

$$Q_{out} = 18000 \text{ kJ}$$

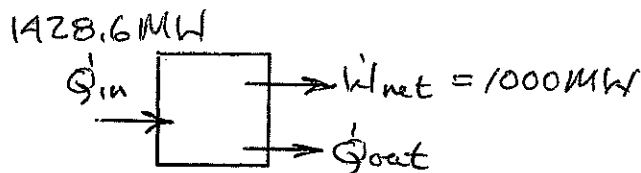
4.82 A heat power cycle with an efficiency of 35% receives 1500 MW of heat. Determine the net power produced in MW.



$$\eta_{th} = 0.35 = \frac{\dot{W}_{net}}{1500}$$

$$\dot{W}_{net} = 525 \text{ MW}$$

4.83 Refer to Example 4. 11. Determine the heat leaving the power plant. If water is used for cooling, receiving the heat from the power plant, and it increases from 15 C to 25 C, what is the required flow rate in cubic meters per second?



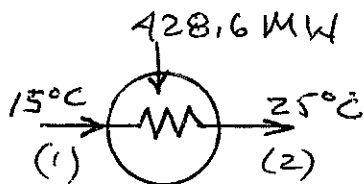
$$\dot{Q}_{in} + \dot{Q}_{out} = \dot{W}_{net}$$

$$1428.6 + \dot{Q}_{out} = 1000$$

$$\dot{Q}_{out} = -428.6 \text{ MW}$$

The heat out, rejected, goes into a heat exchanger where it is transferred to cooling water,

Open System - cooling water.



There is no work done, the changes in kinetic and potential energies are zero, the pressure and density of the water remain constant, so  $s(P/e) = 0$

$$\dot{Q} + \dot{m} \left( u + \frac{P}{\rho} + \frac{ke}{2} + pe \right)_1 = \dot{m} + \dot{m} \left( u + \frac{P}{\rho} + \frac{ke}{2} + pe \right)_2$$

$$\dot{Q} = \dot{m} (u_2 - u_1) = \dot{m} c (T_2 - T_1)$$

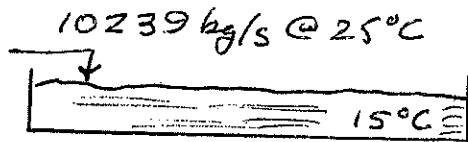
$$428600 \frac{\text{kJ}}{\text{s}} = (\dot{m} \text{ kg/s}) \left( 4.186 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (298 - 288 \text{ K})$$

$$\dot{m} = 10238.9 \text{ kg/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{10238.9 \text{ kg/s}}{1000 \text{ kg/m}^3} = 10.24 \text{ m}^3/\text{s}$$

4.84 The cooling water in Problem 4.83 comes from a lake where the return is mixed. Atmospheric cooling at night maintains a stable temperature. However, the specifications require that the lake must be large enough such that the mixing of the 25°C water into the lake water at 15°C will not cause the lake water to increase in temperature more than 0.5°C in a 24-hour period. How large a volume must the lake be? (The density of water is 1000 kg/m<sup>3</sup>)



The worst case scenario is where the total water increases to 15.5°C in a 24h period,

The inlet water in 24h is

$$m_{in} = (10239 \text{ kg/s})(3600 \text{ s/h})(24 \text{ h/d}) = 8.846 \times 10^8 \frac{\text{kg}}{\text{d}}$$

This is now a mixing problem

$$U_{in} + U_{initial} = U_{final}$$

$$m_{in} c T_{in} + m_{initial} c T_{init} = m_{total} c T_{final}$$

$$(8.846 \times 10^8 \text{ kg}) \left( 4.186 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (298 \text{ K})$$

$$+ (m_{init}) (4.186) (288) = (8.846 \times 10^8 + m_{init}) (288.5) (4.186)$$

$$m_{init} = 1.681 \times 10^{10} \text{ kg}$$

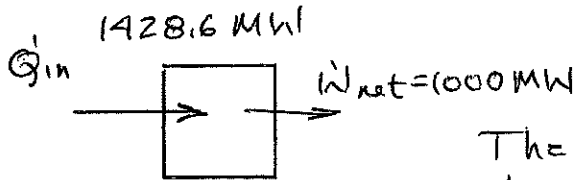
$$m_{in} = 0.088 \times 10^{10} \text{ kg}$$

$$m_{total} = 1.769 \times 10^{10} \text{ kg}$$

$$V = \frac{m}{\rho} = \frac{1.769 \times 10^{10} \text{ kg}}{1.0 \times 10^3 \text{ kg/m}^3}$$

$$V = 1.769 \times 10^7 \text{ m}^3$$

4.85 The power plant in Example 4.11 now uses coal with a 3% sulfur content. Determine the sulfur dioxide produced, with the same scrubber efficiency mentioned in the text, and the tons of coal required each day. If a railroad car holds 86 000 kg, how many carloads of coal are needed per week?



The heat input comes from burning coal

$$\dot{Q}_{in} = 1428600 = \dot{m}_{coal} (27900 \frac{\text{kJ}}{\text{kg}})$$

$$\dot{m}_{coal} = 51,204 \text{ kg/s}$$

Per day  $\dot{m}_{coal} = (51,204 \text{ kg/s}) (3600 \frac{\text{s}}{\text{h}}) (24 \text{ h/d})$

$$\dot{m}_{coal} = 4424051 \text{ kg/d}$$

$$1000 \text{ kg} = 1 \text{ metric ton} \quad \dot{m}_{coal} = 4424 \frac{\text{metric tons}}{\text{d}}$$

$$(4424 \frac{\text{m tons}}{\text{d}}) (7 \frac{\text{d}}{\text{wk}}) (\frac{1}{86 \frac{\text{tons}}{\text{car}}}) = 360 \text{ carloads/wk}$$

The sulfur produced daily is

$$\dot{m}_s = (4424051 \text{ kg/d}) (0.03) (0.02) = 2654 \text{ kg/d}$$

There are 2 kg SO<sub>2</sub> / kg S

$$\therefore = 5308 \text{ kg SO}_2 / \text{day}$$



## Appendix

1. A student received grades of 85, 84, and 91 on her first three chemistry quizzes. What grade must she obtain on her fourth quiz to reach a 90 average?

$$85 + 84 + 91 + x = 4 \times 90 = 360$$

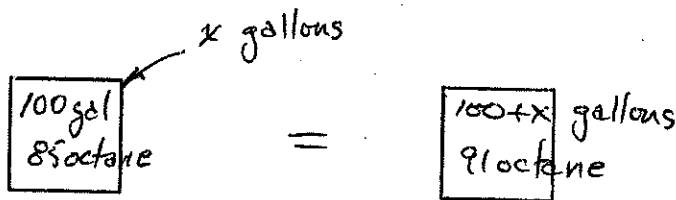
$$x = 100$$

2. A student received grades of 78, 82 and 72 on three tests in physics. The final exam counts as two test grades. What must she score on the final exam to have an average of 80 for the course?

$$78 + 82 + 72 + 2x = 5 \times 80 = 400$$

$$x = 84$$

3. The octane rating of a gasoline is determined by comparing an engine's peak pressure from an actual gasoline mixture to a standard value. The octane rating of a mixture is determined by the volumetric addition of the fuels, thus equal volumes of 80 octane and 100 octane yield a 90 octane mixture. Determine how many gallons of 95 octane fuel must be added to 100 gallons of 85 octane fuel to obtain a 91 octane mixture.

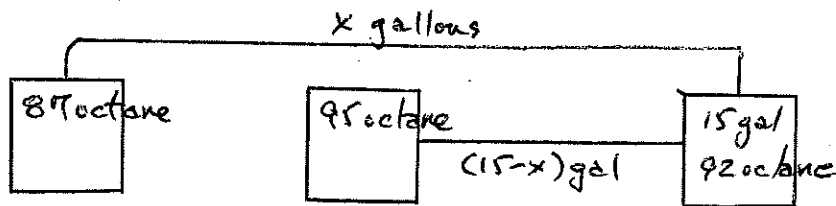


$$x(95) + (100)(85) = (100 + x)(91)$$

$$4x = 600$$

$$x = 150 \text{ gallons}$$

4. The pumps at a service station blend 87 octane with 95 octane gasoline to obtain an octane rating between the two. A customer receives 15 gallons of 92 octane fuel. How many gallons of 87 octane were used?



$$87x + 95(15-x) = 92(15) = 1380$$

$$8x = 45$$

$$x = 5.625 \text{ gal}$$

5. A tank holds 500 kilograms of brine with a salt concentration of 20% by mass. How much water must be evaporated so the concentration rises to 50%?

$$\boxed{\begin{array}{l} 500 \text{ kg} \\ 20\% \text{ salt} \end{array}}$$

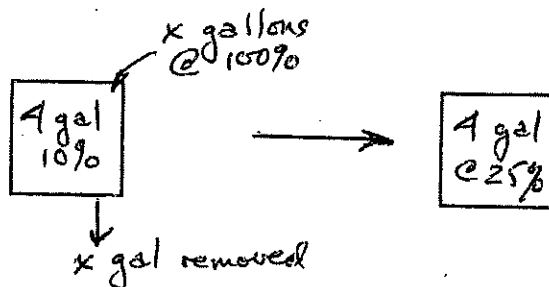
$$\boxed{50\% \text{ salt}}$$

$$(0.2)(500) = 100 \text{ kg salt initially} = 100 \text{ kg final}$$

$$0.5 = \frac{100}{x} \quad x = 200 \text{ kg total}$$

$\therefore$  300 kg evaporated

6. The radiator in an automobile holds 4 gallons of a 10% antifreeze/water mixture. The percentage of antifreeze must be raised to 25% by draining some of the mixture and adding 100% antifreeze. How much mixture must be drained? All percents are on a volume basis.



Antifreeze initial + Antifreeze added = Antifreeze final

$$(4-x)(0.1) + x(1.0) = (4)(0.25) = 1.0$$

$$x = 1 \text{ gal}$$

7. A ceramic clay contains 50% silica, 10% water and 40% other minerals. Determine the percentage of silica on a dry (water-free) basis.

$$\boxed{\begin{array}{l} 50\% \text{ silica} \\ 10\% \text{ H}_2\text{O} \\ 40\% \text{ other} \end{array}}$$

$$\frac{50}{90} = 55.5\%$$

8. Gold has a value of \$12 a gram. A student finds a large gold ore nugget weighing 1000 grams that contains gold and quartz. The density of gold is  $19.3 \text{ g/cm}^3$ , the density of quartz is  $2.5 \text{ g/cm}^3$  and the density of nugget is  $6.5 \text{ g/cm}^3$ . The student is offered \$150 for the nugget; should he accept the offer?

$$\begin{aligned} & \rightarrow 1000 \text{ g} \\ & \rho = 6.5 \text{ g/cm}^3 \end{aligned}$$

$$\rho_{\text{Ag}} = 19.3 \text{ g/cm}^3 \quad 12 \text{ \$/g}$$

$$\rho_{\text{quartz}} = 2.5 \text{ g/cm}^3$$

$$x = \text{grams gold}$$

$$(1000 - x) = \text{grams quartz}$$

$$x(19.3) + (1000 - x)(2.5) = (1000)(6.5)$$

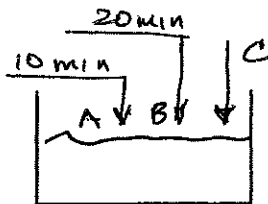
$$16.8x = 4000$$

$$x = 238.1 \text{ g @ } \$12/\text{g}$$

$$\$2857 \text{ — value of ore}$$

Don't accept offer

9. A tank may be filled using pipe A or B with times of 10 and 20 minutes, respectively. It takes only 5 minutes to fill the tank when pipe C is used simultaneously with pipes A and B. How long does it take to fill the tank using only pipe C?



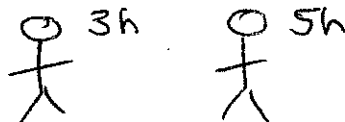
$$5 \left( \frac{1}{10} + \frac{1}{20} + \frac{1}{C} \right) = 1.0$$

$$0.1 + 0.05 + x = 0.2$$

$$x = 0.05$$

$$C = 20 \text{ min}$$

10. Two workers can assemble a device in 3 hours and 5 hours, respectively. How long would it take to assemble the device if they worked together?

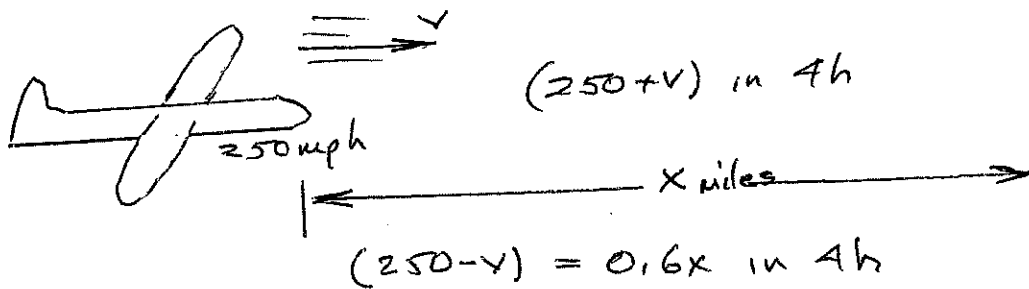


$$t \left( \frac{1}{3} + \frac{1}{5} \right) = 1.0$$

$$t(0.533) = 1.0$$

$$t = 1.88 \text{ h}$$

11. An airplane flies with a velocity of 250 mph when there is no wind. In the flying with the wind it travels a certain distance in 4 hours. However, in flying against the wind it can only travel 60% of that distance. What is the wind's velocity?



$$x = (250 + v)(4)$$

multiply by 0.6

$$0.6x = (2.4)(250 + v) \quad (a)$$

Also

$$0.6x = (250 - v)(4) \quad (b)$$

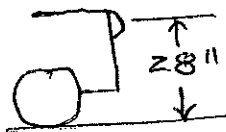
Equate (a) and (b)

$$(2.4)(250 + v) = 4(250 - v)$$

$$6.4v = 400$$

$$v = 62.5 \text{ mph}$$

12. A state's automobile inspection program checks headlight alignment with the specifications that the light beam drop cannot be greater than 2 inches for each 25 feet in front of the car. Suppose the headlights on your car are 28 inches above the ground and that they meet the 2 inch drop requirement, what is minimum distance in front of the car they can illuminate? If you are driving at 50 mph, how long does it take to travel that distance?



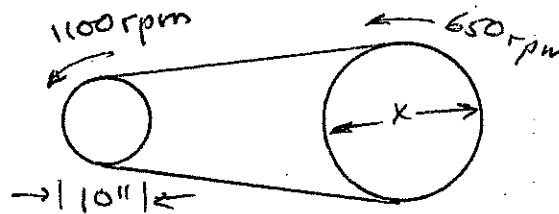
$$28" \times \frac{25 \text{ ft}}{2"} = 350 \text{ feet}$$

$$\left( \frac{50 \text{ mi}}{\text{h}} \right) \left( \frac{88 \text{ ft}}{\text{min}} (\text{mph}) \right) = 4400 \text{ ft/min}$$

$$\frac{4400 \text{ ft/min}}{60 \text{ s/min}} = 73.3 \text{ ft/sec}$$

$$\frac{350 \text{ ft}}{73.3 \frac{\text{ft}}{\text{sec}}} = 4.77 \text{ sec}$$

13. A tractor has a belt pulley diameter of 10 inches operating a 1100 revolutions per minute (rpm). The pulley is connected to another machine that needs to operate at 650 rpm. What size pulley should be used on the machine?



$$\frac{1100}{650} = \frac{x}{10}$$

$$x = 16.92 \text{ inches}$$

14. The following table lists the amount of grain and hay that a steer is fed to produce the desired weight gain. The grain costs 0.15 \$/pound and the hay costs 0.06 \$/pound. Determine the lowest cost combination.

**Combinations of grain and hay to produce satisfactory weight gain in a steer.**

Pounds of Hay	Pounds of Grain
1000	1316
1100	1259
1200	1208
1300	1162
1400	1120
1500	1081
1600	1046
1700	1014
1800	984
1900	957

*Use a spreadsheet to solve for least expensive combination.*

1000	1316	257.4
1100	1259	254.85
1200	1208	253.2
1300	1162	252.3
1400	1120	252 ← total minimum cost
1500	1081	252.15
1600	1046	252.9
1700	1014	254.1
1800	984	255.6
1900	957	257.55

15. Inventory turnover is the ratio of the cost of goods sold to the average inventory value,  

$$\text{Inventory value} = \frac{\text{Cost of goods sold}}{\text{Average inventory value}}$$
 The XYZ Office Supply Company started the year with an inventory worth \$28,532 and ended the year with an inventory worth \$33,124. During the year the business sold \$264,845 worth of office supplies. What is the inventory turnover ratio? Is a high ratio desired or not? Why?

$$\text{Inv}_T = \frac{264845}{30828} = 8.59$$

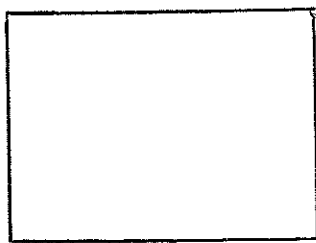
Ave inventory value

$$\frac{28532 + 33124}{2} = 30828$$

High ratio — more product sold & hence greater profit potential

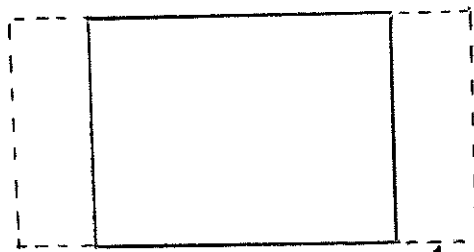
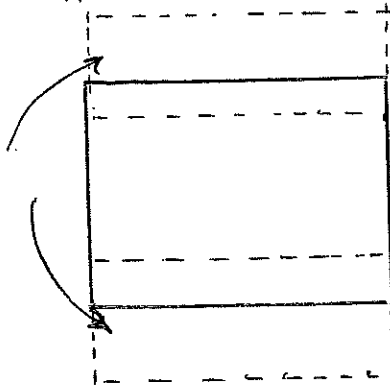
16. The standard aspect ratio (width to height) for a television picture is 4:3. The program director has scheduled to broadcast a wide-screen movie that was filmed with an aspect ratio of 2:1. Suppose that the showing will broadcast the full width of the movie, while maintaining the aspect ratio of 2:1. What would the result look like on the television screen? Illustrate with a sketch. What would happen if the movie were broadcast at full height?

Aspect ratio  $\frac{W}{H} = \frac{4}{3}$



$$\frac{W}{H} = \frac{2}{1} * \frac{3}{3} = \frac{6}{3}$$

Missing picture



Missing picture

17. You are managing a mail-order business that has 5 workers assigned to process and package orders. These workers are packaging an order for 2500 parts which needs to be mailed by the end of the 8 hour shift. So far they have packaged 900 parts during the first four hours. It is evident that they are not going to be able to finish the order during the time remaining without additional help. How many additional workers are required to complete the task during the last four hours?

$$\frac{900 \text{ parts}}{4 \text{ h}} = \frac{225 \text{ parts/h}}{5 \text{ people}} = 45 \text{ parts/h/person}$$

$$2500 - 900 = \frac{1600 \text{ parts remaining}}{4 \text{ h}} = 400 \text{ parts/h}$$

$$400 = 45x \text{ where } x \text{ are \# of people}$$

$$x = 8.88 \text{ or } 9 \text{ total}$$

∴ 4 more workers

18. During the first year of operation a clinic treated a total of 4916 patients and gave 624 of them flu immunizations. This year the clinic is treating more patients, 3384, during the first six months and has given 487 flu immunizations. You need to order supplies; estimate the number of patients and flu immunizations for the rest of the year.

$$\text{The first year The flu shots/patient} = \frac{624}{4916} = 0.1269$$

$$\text{The second half year flu shot/patient} = \frac{487}{3384} = 0.1439$$

With no additional information, assume linear variation for the second six months.

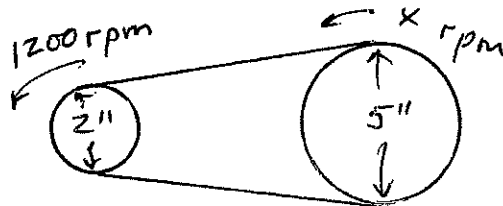
∴ total patients for rest of year is 6776  
or 3384 additional ones; 487 additional immunizations

19. A water treatment plant deals with very large volumes of water and very small concentrations of chemicals used in the water purification processes. In one process a chemical must be added in the amount of 0.7 parts per million. The treatment facility has holding tanks that contain 850,000 gallons. How many gallons of chemical should be added to the holding tank to obtain the desired concentration?

$$0.7 \text{ parts/million} \quad 850,000 \text{ gallons}$$

$$(0.85 \text{ million gallons})(0.7) = \underline{0.595 \text{ gallons}}$$

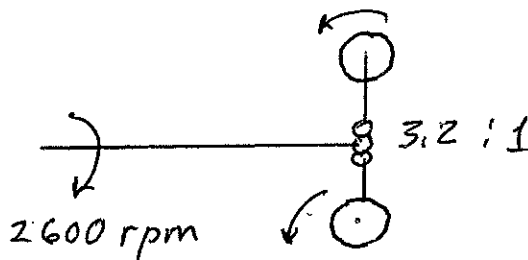
20. A two-inch diameter pulley is mounted on a motor shaft that rotates at 1200 revolutions per minute (rpm). This pulley is connected to a five-inch diameter pulley mounted on the shaft of a fan. Determine the fan's rpm.



$$\frac{2}{5} = \frac{x}{1200}$$

$$x = 480 \text{ rpm}$$

21. The gears in a transmission are often described in ratios, such as 3:1, which means that the drive gear rotates three revolutions while the driven gear rotates one revolution. In a certain automobile the final transmission and differential gear reduction is 3.2:1. The engine crankshaft rotates at 2600 rpm, how fast does the drive axle rotate? If tires with a 26 inch diameter are attached to the axles, what is speed of the car at this rpm?



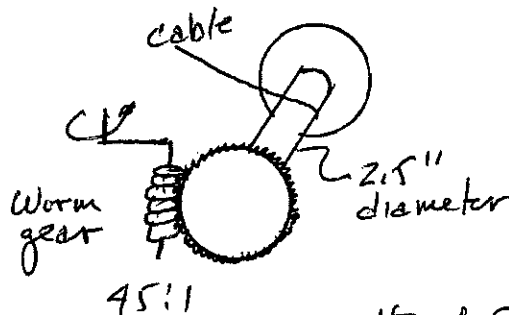
$$\text{Wheel rpm} = \frac{2600}{3.2} = 812.5 \text{ rpm}$$

$$\left(812.5 \frac{\text{rev}}{\text{min}}\right) \left(\pi\right) \left(\frac{26 \text{ ft}}{12}\right) = 5530.5 \frac{\text{ft}}{\text{min}}$$

$$\frac{5530.5}{88} = 62.8 \text{ mph}$$



22. A worm gear is used in manual winches, such as the one shown below. The worm gear has a ratio of 45:1, indicating that 45 turns of the hand crank are needed to rotate the large gear once. Suppose that the drum attached to the large gear has a diameter of 2.5 inches and is used to reel in 12 feet of cable. In a test, you find you can rotate the hand crank at 40 turns per minute. How many minutes will it take to reel in the cable?



$$\frac{\text{Distance}}{\text{rev}} = \pi D = \pi \frac{2.5}{12}$$

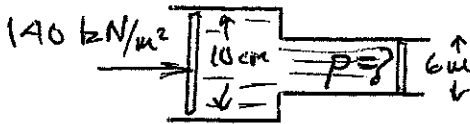
$$= 0.6545 \text{ ft/rev}$$

$$\# \text{ of Rev's} = \frac{12 \text{ ft}}{0.6545} = 18.33 \text{ rev of } 2.5" \text{ drum}$$

$$45 * 18.33 = 825 \text{ rev of worm gear}$$

$$\frac{825 \text{ rev}}{40 \text{ rev/min}} = 20.6 \text{ min}$$

23. The pressure in a system may be increased by using two pistons of different diameters connected to a common push rod. For the pistons to remain in static equilibrium, the force on each piston surface (pressure times area) must be equal. The larger piston has a diameter of 10 centimeters and the smaller piston has a diameter of 6 centimeters and the pressure acting on the larger piston is  $140 \text{ kN/m}^2$ , what is the pressure acting on the smaller diameter piston? If the diameter of the larger piston increases by 20%, everything else remaining constant, what is the pressure on the smaller piston?



For static equilibrium

$$(F_x)_{10 \text{ cm}} = (F_x)_{6 \text{ cm}}$$

$$(p A)_{10} = (p A)_6$$

$$\left(140 \frac{\text{kN}}{\text{m}^2}\right) \left(\frac{\pi}{4} (0.1)^2 \text{ m}^2\right) = \left(p \left(\frac{\pi}{4} (0.06)^2\right)\right)$$

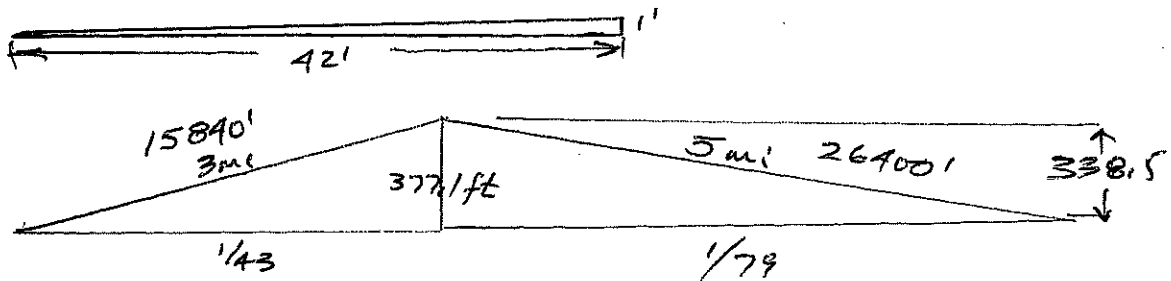
$$a) \quad p = 388.9 \text{ kN/m}^2$$

If diameter of piston is 12 cm

$$(140)(0.12)^2 = p(0.06)^2$$

$$p = 560 \text{ kN/m}^2 \text{ an increase of } 44\%$$

24. The steepness of a railroad track over a three mile grade is reported as a rising grade of 1 in 43, meaning that it rises one foot for every 42 feet in track. Following the rise, the track now descends 5 miles with a descending grade of 1 in 79. Is the elevation at the end of the 5 mile descent less than, greater than or equal to the elevation at the start of three mile ascent?

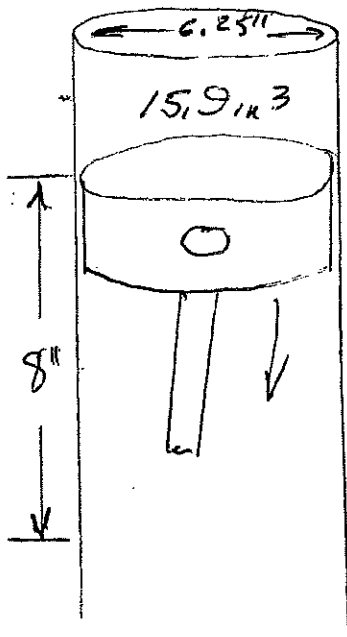


$$\frac{15840}{42} = 377.1$$

$$\frac{26400}{79} = 338.5'$$

The track is higher after the descent than it was originally by  $(377.1 - 338.5)$  38.6 feet.

25. The compression ratio of an internal combustion engine is described in terms of volume ratios, the volume at the beginning of the compression divided by the volume remaining at the end of compression. These compression ratios are expressed as 8:1 or 15:1. A diesel engine has a cylinder of with a 6.25 inch bore and an 8 inch stroke. The volume left at the top of compression stroke is 15.9 cubic inches. Determine the engine's compression ratio.

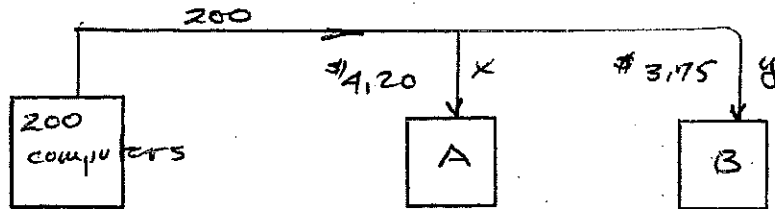


$$V_{\text{swept}} = \frac{\pi}{4} (6.25)^2 (8) = 245.4 \text{ in}^3$$

$$V_{\text{total}} = 245.4 + 15.9 = 261.3 \text{ in}^3$$

$$r = \frac{261.3}{15.9} = 16.4$$

26. A computer manufacturer ships 200 computers to two different stores, A and B. It costs \$4.50 to ship to A and \$3.75 to ship to B. The total shipping invoice was \$806.25. How many computers were shipped to each location?



$$200 = x + y$$

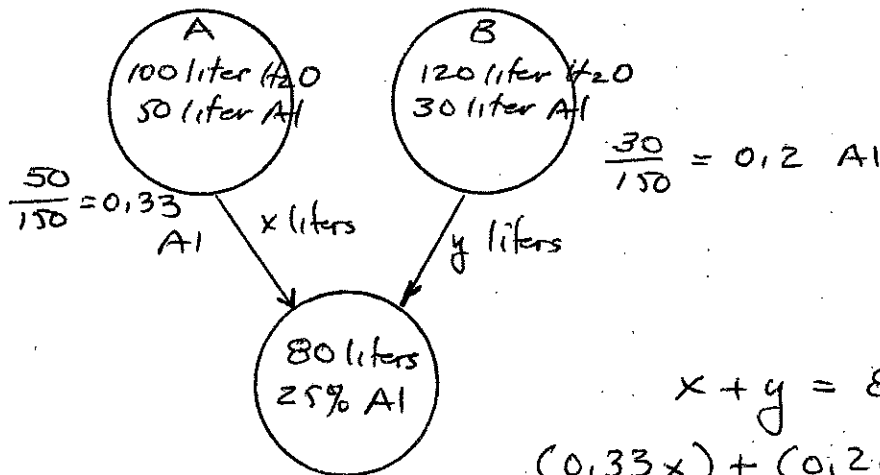
$$806.25 = x(4.20) + y(3.75)$$

$$806.25 = 4.2x + 3.75(200 - x)$$

$$x = 125 \text{ to A}$$

$$y = 75 \text{ to B}$$

27. Tank A contains a mixture of 100 liters of water and 50 liters of alcohol while tank B has 120 liters of water and 30 liters of alcohol. How many liters should be taken from the tanks to create an 80 liter mixture that is 25% alcohol by volume?



$$x + y = 80$$

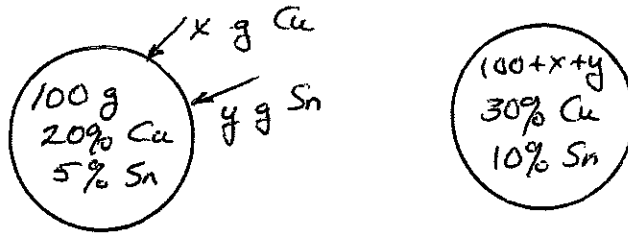
$$(0.33x) + (0.2y) = (0.25)(80)$$

$$(0.33x) + (0.2)(80 - x) = 20$$

$$x = 30.77 \text{ liters}$$

$$y = 49.23 \text{ liters}$$

28. In a material science laboratory a 100 gram alloy is found to contain 20% copper and 5% tin by weight. How many grams of pure copper and pure tin must be added to this alloy to produce another alloy that is 30% copper and 10% tin?



$$\frac{\text{Copper}}{\text{Initial}} + \frac{\text{Copper}}{\text{Added}} = \frac{\text{Copper}}{\text{Final}}$$

$$0.2(100) + x(1.0) = 0.3(100+x+y)$$

$$0.7x - 0.3y = 10 \quad (a)$$

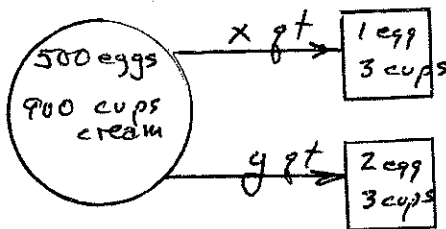
$$\begin{array}{l} \text{Tin} \\ \text{Balance} \end{array} \quad (0.05)(100) + y(1.0) = 0.1(100+x+y)$$

$$-0.1x + 0.9y = 5 \quad (b)$$

$$\text{Solve (a)+(b)} \quad 0.7x - 0.3y = 10$$

$$y = 7.5 \text{ g} \quad x = 17.5 \text{ g}$$

29. You are coordinating the ice cream making at a state agricultural fair where two types of homemade ice cream will be available. There are two main ingredients, eggs and cream, and there are 500 eggs and 900 cups of cream available. Plain Vanilla requires one egg and three cups of cream per quart while French Vanilla needs two eggs and three cups of cream per quart. Determine the number of quarts of each variety that should be made to use up all the ingredients.



$$\begin{array}{l} \text{Egg} \\ \text{Balance} \end{array} \quad 1x + 2y = 500 \quad (a)$$

$$\begin{array}{l} \text{Cream} \\ \text{Balance} \end{array} \quad 3x + 3y = 900$$

$$3x + 6y = 1500$$

$$3y = 600$$

$$y = 200 \text{ qts}$$

$$x = 100 \text{ qts}$$

Multiply (a) by 3

30. In intramural sports, one dorm has won a total of 12 games this year, some in volleyball others in soccer. There is a rating system where each win in volleyball counts as two points and each win in soccer counts as four points. The dorm has a total of 38 points, how many soccer and volleyball games did it win?



12 games won  
 Volleyball 2pts/win  $x$   
 Soccer 4pts/win  $y$

$$x + y = 12 \quad (a)$$

$$2x + 4y = 38$$

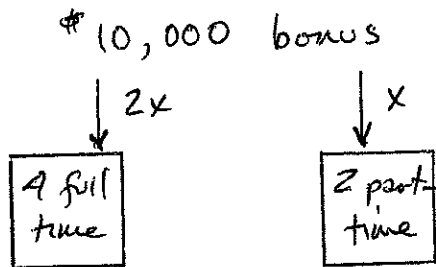
Multiply (a) by 2 and subtract

$$2x + 2y = 24$$

$$2y = 14$$

$$y = 7 \text{ soccer} \quad x = 5 \text{ volleyball}$$

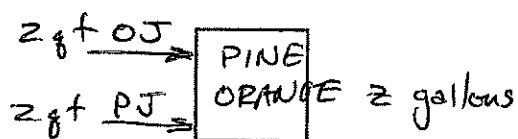
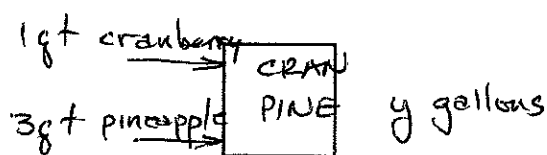
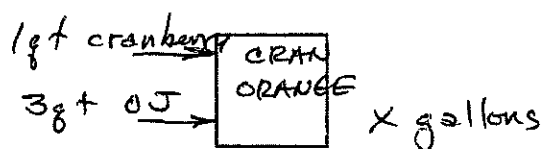
31. As a project manager, you are responsible for allocating a \$10,000 bonus between 4 full-time and 2 part-time employees. You decide on the algorithm that the full-time employees will receive an amount which is twice that of the part-timers. What is the amount the full-timers and part-timers receive?



$$2x + 4(2x) = 10000$$

$$x = \$1000$$

32. The Tastee Beverage Company makes three types of juice drinks: CranOrange, using one quart of cranberry juice and three quarts of orange juice; CranPine, using one quart of cranberry juice and three quarts of pineapple juice; and PineOrange, using two quarts of pineapple and two quarts of orange juice per gallon. Each day the company uses 350 quarts of cranberry juice, 800 quarts of orange juice and 650 quarts of pineapple juice. How many gallons of each blend are produced daily to uniquely use the above amounts of juice?



350 gts CJ }  
800 gts OJ } daily  
650 gts PJ }

CRANBERRY BALANCE	$1x + 1y$	$= 350$	$x + y = 350$
OJ BALANCE	$3x + 0y + 2z$	$= 800$	$3x + 2z = 800$
PJ BALANCE	$0x + 3y + 2z$	$= 650$	$3y + 2z = 650$

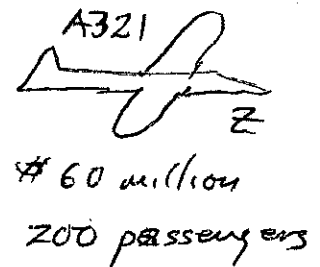
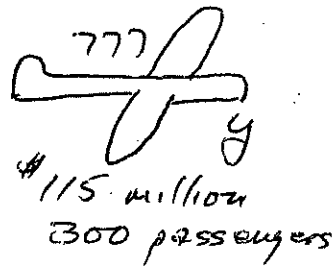
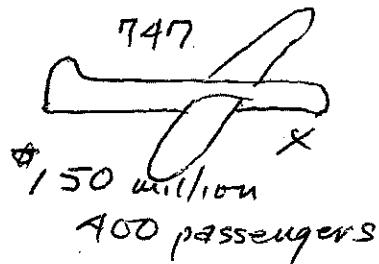
Substitute and solve for x, y, z

$x = 200$  gal Cran Orange

$y = 150$  gal Cran Pine

$z = 100$  gal Pine Orange

33. As President of ExpressAir, you are considering the purchase of additional airplanes to expand your company's capacity by 2000 seats. A mix of aircraft type is desired because of routing and the following information is known: Boeing 747's cost \$150 million each and carry 400 passengers; Boeing 777 cost \$115 million each and carry 300 passengers and Airbus A321 cost \$60 million and carry 200 passengers. The routes indicate that a wise mix of aircraft would be equal numbers of 747's and 777's. The total budget available is \$710 million. How many of each aircraft can be purchased and satisfy the seating increase?



$$(a) \quad x = y$$

$$400x + 300y + 200z = 2000$$

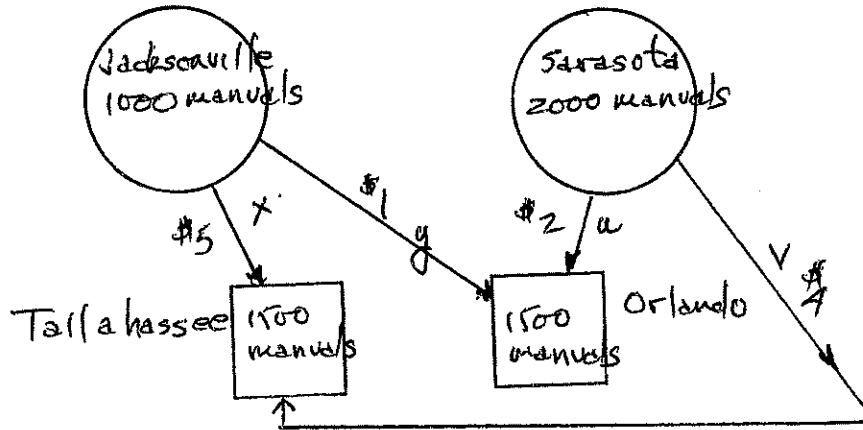
$$(b) \quad 4x + 3y + 2z = 20$$

$$(c) \quad 150x + 115y + 60z = 710$$

Solve (a), (b), (c) simultaneously. Sub (a) in

$$(b) \text{ \& } (c) \text{ yielding } z = 3, x = 2, y = 2$$

34. A company maintains two distribution warehouses, one in Jacksonville and the other in Sarasota. The warehouses supply software manuals to two retail outlets, one in Orlando and the other in Tallahassee. The Jacksonville warehouse has 1000 manuals and the one in Sarasota has 2000 manuals. Each retail store orders 1500 manuals. It costs \$1 to ship a manual from Jacksonville to Orlando and \$2 to ship one from Sarasota to Orlando. It costs \$5 to ship a manual from Jacksonville to Tallahassee and \$4 to ship a manual from Sarasota to Tallahassee. For a budget of \$9000, how many manuals should be shipped from each warehouse to satisfy each store's requirements.



$$u + v = 2000$$

$$5x + y + 2u + 4v = 9000$$

$$x + v = 1500$$

$$y + u = 1500$$

Solve these equations for  $x, y, u, v$

$$v = u = 1000$$

$$x = y = 500$$



35. Solve Example 4 using matrix methods.

Create a matrix of coefficients						
			Matrix A			Matrix D
		0.25	0.2	0.4		22
		0.4	0.3	0.2		28
		0.3	0.1	0.2		18
Find the inverse of Matrix A						
		-2.85714	0	5.714286		
		1.428571	5	-7.85714		
		3.571429	-2.5	0.357143		
Multiply the inverse Matrix by Matrix D						
Yielding the solution to the problem						
			40 kg	x		
			30 kg	y		
			15 kg	z		

36. A service station sells three grades of gasoline, regular, premium and super. One day the station sold 150 gallons of regular, 400 gallons of premium and 130 gallons of super for a total of \$909. The next day it sold 170 gallons of regular, 380 gallons of premium and 150 gallons of super for \$931. The price difference between super and regular is one-half the difference between premium and regular. Determine the cost per gallon for each grade of gasoline.

$$\begin{array}{ccc}
 x \text{ \$/gal} & y \text{ \$/gal} & z \text{ \$/gal} \\
 \boxed{\begin{array}{c} \text{REG} \\ 150 \text{ gal} \end{array}} & \boxed{\begin{array}{c} \text{PREM} \\ 400 \text{ gal} \end{array}} & \boxed{\begin{array}{c} \text{SUPER} \\ 130 \text{ gal} \end{array}} & = \$909
 \end{array}$$

$$\begin{array}{ccc}
 170 \text{ gal} & 380 \text{ gal} & 150 \text{ gal} & = 931
 \end{array}$$

$$150x + 400y + 130z = 909$$

$$170x + 380y + 150z = 931$$

$$z - x = \frac{1}{2}(y - x)$$

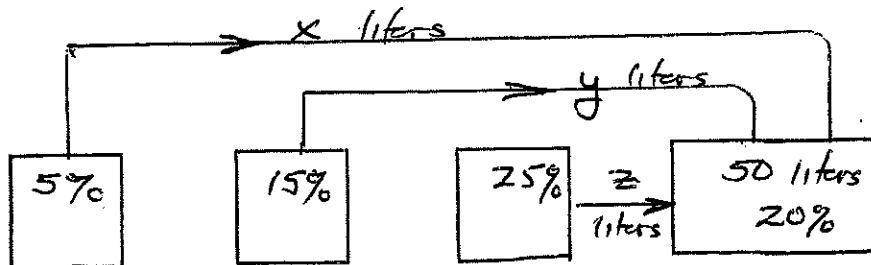
Solve

$$x = 1.17/\text{gal}$$

$$y = 1.41/\text{gal}$$

$$z = 1.29/\text{gal}$$

37. A chemical engineer has three salt solutions available, 5%, 15% and 25%, to make 50 liters of a 20% solution. There is much more 5% solution available, so a requirement is to use twice as much 5% solution as the 15% solution. Determine the amount of each salt solution that is used to make the mixture.



$$x + y + z = 50$$

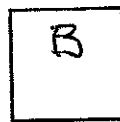
$$0.05x + 0.15y + 0.25z = (0.2)(50) = 10$$

$$x = 2y$$

Solve for  $x, y, z$

$y = 5$  liters 15%  
 $x = 10$  liters 5%  
 $z = 35$  liters 25%

38. A manufacturing company produces two products, I and II, that require time on machines A and B. Product I requires 1 hour on A and 2 hours on B, while product II requires 3 hours on A and 1 hour on B. The company is open 16 hours per day, with the machines operating 15 hours per day. What is the number of each product that can be produced daily?



Product I	1 hr A, 2 hr B	x
Product II	3 hr A, 1 hr B	y

Machines operate 15 h/day

Machine A time  $1x + 3y = 15$

Machine B time  $2x + 1y = 15$

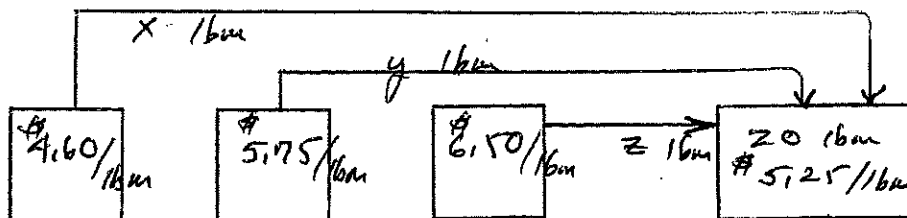
Solve

$$y = 3$$

$$x = 6$$

6 Product I    3 Product II

39. Three grades of resin are available which may be mixed together to form a fourth resin. The costs of the initial resins are \$4.60, \$5.75 and \$6.50 per pound. The mixture value will be \$5.25 per pound and 20 pounds are needed. In addition, the amount of the least expensive resin should be equal to the total amount of the other two. Determine the amount of each resin needed.



$$x + y + z = 20$$

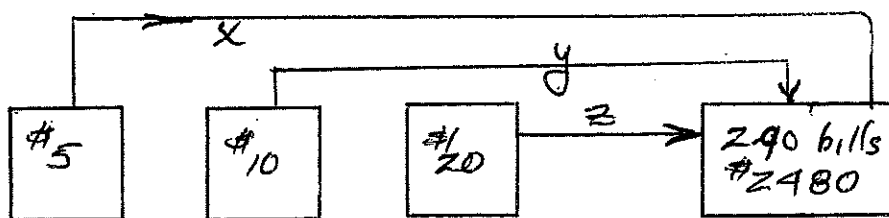
$$4.60x + 5.75y + 6.50z = (20)(5.25) = 105$$

$$x = y + z$$

Sub value of  $x$  into previous equations, solving for  $y + z$ , then  $x$ .

$$z = 2 \text{ lbm} \quad y = 8 \text{ lbm} \quad x = 10 \text{ lbm}$$

40. An engineering club holds a benefit party and collects a total of \$2480 consisting of five, ten and twenty dollar bills. The total number of bills is 290. The value of the total number of tens is \$60 more than the value of the total number of twenties. Determine the number of each type of bill the club has.



$$x + y + z = 290$$

$$5x + 10y + 20z = 2480$$

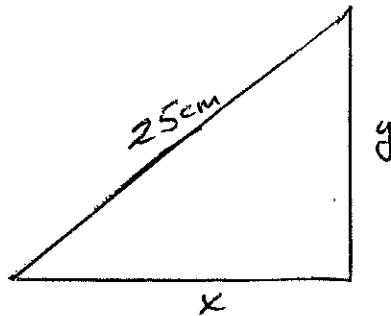
$$10y = 60 + 20z$$

$$y = 6 + 2z$$

Sub  $y$  equation in first two, solving for  $x + z$ .

$$z = 40 \text{ (20)} \quad x = 164 \text{ (5)} \quad y = 86 \text{ (10)}$$

41. A right triangle is formed from a wire 60 cm long. The triangle's hypotenuse is 25 cm. Find the length of the other two sides.



$$x = 20 \text{ cm}$$

$$y = 15 \text{ cm}$$

$$x + y + 25 = 60$$

$$x + y = 35$$

$$x^2 + y^2 = 25^2 = 625$$

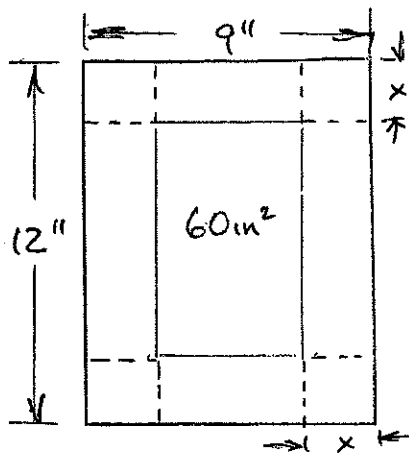
$$x^2 + (35 - x)^2 = 625$$

$$x^2 + 1225 - 70x + x^2 = 625$$

$$x^2 - 35x + 300 = 0$$

$$(x - 20)(x - 15) = 0$$

42. A student is given a 9 inch by 12 inch piece of paper and is construct an open box by cutting equal squares from each of the corners of the paper and then folding up the sides. The base areas should be 60 square inches. Find the length of the sides of the squared that are removed.



$$(12 - 2x)(9 - 2x) = 60$$

$$108 - 42x + 4x^2 = 60$$

$$2x^2 - 21x + 24 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x =$$

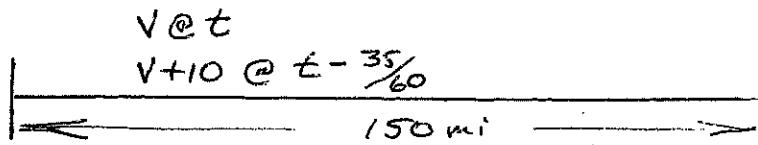
$$2a$$

$$x = \frac{+21 \pm \sqrt{21^2 - (4)(2)(24)}}{4}$$

$$x = 9.19, 1.3$$

The only physically possible value is 1.3

43. A student is driving home, a distance of 150 miles, for the weekend. From previous experience, the student knows that increasing the average speed 10 miles/hour could reduce the time of the trip by 35 minutes. What is the actual average speed?



$$V \times t = 150 \quad t = 150/V$$

$$(V+10)\left(t - \frac{35}{60}\right) = 150$$

$$(V+10)\left(150/V - 0.5833\right) = 150$$

$$(V+10)(150 - 0.5833V) = 150V$$

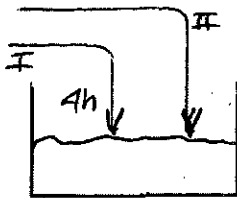
$$-0.5833V^2 - 5.833V + 1500 = 0$$

$$V^2 + 10V - 2571.6 = 0$$

$$V = \frac{-10 \pm \sqrt{10^2 - (4)(1)(-2571.6)}}{2}$$

$$V = +45.95 = 46 \text{ mph} \quad \text{or} \quad -56 \text{ mph not possible}$$

44. Two pipes, I and II, can be used to fill a tank. Pipe I fills the tank in four hours. If pipe II is used by itself, it takes 3 hour longer than if both pipes are used simultaneously. Determine the time it takes to fill the tank with pipe II.



$$t_{II} = 3 + t_{I+II}$$

$$t \left( \frac{1}{4} + \frac{1}{t+3} \right) = 1.0$$

$$t \left[ \frac{t+3+4}{4(t+3)} \right] = 1.0$$

$$t^2 + 7t = 4t + 12 \quad t^2 + 3t - 12 = 0$$

$$t = \frac{-3 \pm \sqrt{3^2 - (4)(1)(-12)}}{2} = +2.275, -5.275$$

$$t = 2.275 \text{ h} \quad \text{only possible value}$$

45. Assume the annual rate of inflation is 5%. Determine how long it will take for prices to double if they rise in proportion to inflation.

$$\begin{aligned}
 i &= 5\% & I &= I_0(1+i)^n \\
 & & 2I_0 &= I_0(1.05)^n \\
 & & \ln(2) &= n \ln(1.05) \\
 & & n &= 14.2 \text{ years}
 \end{aligned}$$

46. The half-life of radioactive carbon 14 is 5700 years. After a plant or animal dies, the level of carbon 14 decreases as the radioactive carbon disintegrates. The decay of radioactive material is given by the relationship,  $A = A_0 e^{-kt}$ , where  $A_0$  is the initial amount of material at time zero and  $t$  represent the time measured from time zero in years. For carbon 14,  $k = 1.216 \times 10^{-4}$  years. Samples from an Egyptian mummy show that the carbon 14 level is one-third that found in the atmosphere. Determine the approximate age of the mummy.

$$\begin{aligned}
 (C_{14})_{t_{\text{half}}} &= 5700 \text{ yr} & k &= 1.216 \times 10^{-4} \text{ yr} \\
 A &= A_0 e^{-kt} \\
 \frac{1}{3} A_0 &= A_0 e^{-kt} \\
 \ln(0.333) &= -kt \ln(e) = -kt \\
 -1.098612 &= -(1.216 \times 10^{-4})(t) \\
 t &= 9109 \text{ years}
 \end{aligned}$$

47. Paint from cave drawings in France indicate a carbon 14 level 15% of that found in the atmosphere. Determine the approximate age of the drawings.

$$\begin{aligned}
 A &= A_0 e^{-kt} \\
 0.15 A_0 &= A_0 e^{-kt} & \ln(0.15) &= -kt \ln(e) \\
 -1.89712 &= -(1.216 \times 10^{-4})(t) \\
 t &= 15601 \text{ years}
 \end{aligned}$$

48. The amount of a certain chemical,  $A$ , that will dissolve in solution varies exponentially with the Celsius temperature,  $T$ , according to the equation  $A = 10 e^{0.01T}$ . Determine the temperature that allows 15 grams of chemical to dissolve.

$$\begin{aligned}
 A &= 10 e^{0.01T} \\
 15 &= 10 e^{0.01T} \\
 \ln(1.5) &= (0.01)(T) \ln(e) \\
 T &= \frac{0.405465}{0.01} = 40.5^\circ\text{C}
 \end{aligned}$$

49. Newton's law of cooling describes the cooling or heating of an object by a fluid (liquid or gas). The temperature variation with time is given by the equation  $T(t) = T_0 + A e^{-kt}$ , where  $A$  is a constant equal to 100,  $k$  is a constant equal to 0.1,  $t$  is the time in minutes and  $T_0$  is the surrounding fluid temperature. Determine the time it will take a cup of hot coffee to cool to 30° C in a room at 20° C.

$$T(t) = T_0 + A e^{-kt}$$

$$30 = 20 + 100 e^{-0.1t}$$

$$10 = 100 e^{-0.1t}$$

$$\ln(0.1) = -0.1t \quad \ln(e) = -0.1t$$

$$t = 23 \text{ minutes}$$

$A = 100$   
 $k = 0.1$   
 $T = 30^\circ\text{C}$   
 $T_0 = 20^\circ\text{C}$

50. Plutonium 239 decays at a rate of 0.00284% per year. If the initial sample size of P-239 is 10 grams, how much will remain as P-239 after 20,000 years?

$$I = I_0 e^{-(0.0000284)(20,000)}$$

$$I_0 = 10$$

$$I = 5.66 \text{ grams}$$

51. A strain of bacteria is reproducing continuously at a rate of 0.31% per minute. A culture with 1000 organisms will double in size in what amount of time?

$$A = P e^{kt}$$

$$2000 = (1000) e^{(0.0031)(t)}$$

$$\ln(2) = 0.0031t \quad \ln(e) = 1$$

$$t = 223.6 \text{ min}$$

52. The Richter scale is used to measure the intensity of earthquakes and is given by the formula  $R = 0.667(\log E - 4.4)$  where  $E$  is the energy released in an earthquake in joules. The San Francisco earthquake of 1906 registered 8.2 on the Richter scale and one in 1989 measured 7.1. What is the percentage of energy released in the 1989 earthquake compared to one in 1906?

$$R = 0.667(\log E - 4.4)$$

$$8.2 = 0.667 \log E - (0.667)(4.4)$$

$$1906 \quad E = 4.943 \times 10^{16}$$

$$7.1 = 0.667 \log E - (0.667)(4.4)$$

$$1989 \quad E = 1.109 \times 10^{15}$$

$$\% E_{1989} = \frac{1.109 \times 10^{15}}{4.943 \times 10^{16}} = \frac{1.109}{49.43} = 0.022 \quad 2.2\%$$

53. The decibel level of sound from a stereo set decreases with distance according to relationship

$$D = 10 \log \left( \frac{320 \times 10^7}{r^2} \right)$$

Determine the decibel rating at 5, 10 and 15 feet. Express the relationship in the form  $D = a + b \log r$ .

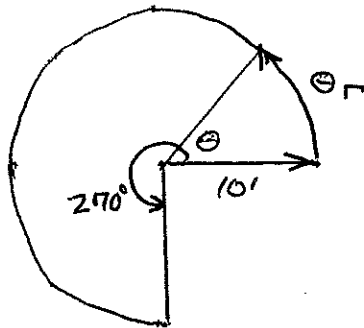
$$Db = 10 \log \left( \frac{A}{r^2} \right) = 10 [\log A - \log r^2]$$

$$Db = 10 [\log A - 2 \log r]$$

$$Db = 10 \log (320 \times 10^7) - 20 \log r$$

$$Db = 95,05 - 20 \log r$$

54. A plot of land is a 270 degree sector with a 10 foot radius. Determine the area..



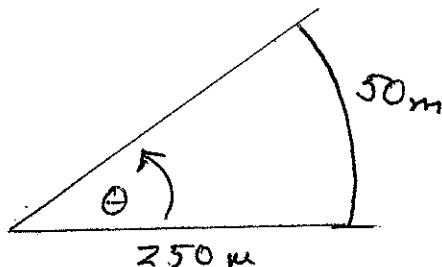
$$A = \left( \frac{\theta r}{2} \right) (r) = \frac{\theta r^2}{2}$$

Express  $\theta$  in radians

$$\frac{\pi}{180} \times 270 = 4,712$$

$$A = \frac{(4,712)(10^2)}{2} = 235,6 \text{ ft}^2$$

55. A curve along a highway is a circular arc 50 m long with a radius of curvature of 250 m. How many degrees does the highway change its direction along the arc?



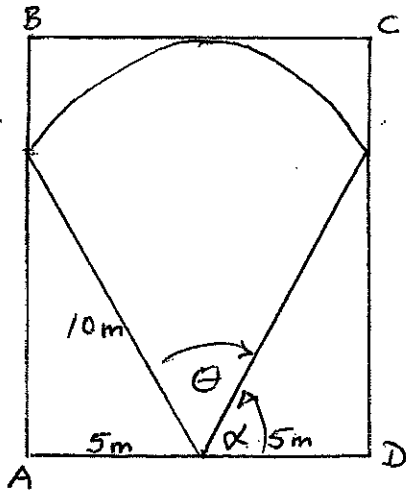
$$50 = r \theta$$

$$\theta = \frac{50}{250} = 0,2 \text{ radians}$$

$$\theta = \left( \frac{180}{\pi} \right) (0,2) = 11,4^\circ$$



56. Find the area of the sector inside the square ABCD.



$$A = \frac{\theta r^2}{2}$$

$$\cos \alpha = \frac{5}{10} = 0.5$$

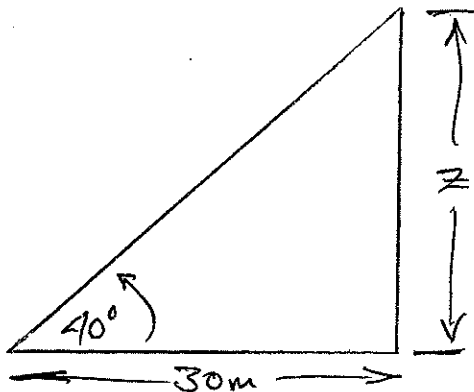
$$\alpha = 60^\circ$$

$$\therefore \theta = 60^\circ$$

$$\theta = \frac{\pi}{180} \times 60 = 1.047 \text{ rad.}$$

$$A = \frac{(1.047)(10^2)}{2} = 52.36 \text{ m}^2$$

57. The angle of elevation to the top of a flagpole is 40 degrees from a point 30 m from the base of the pole. What is the height of the pole?

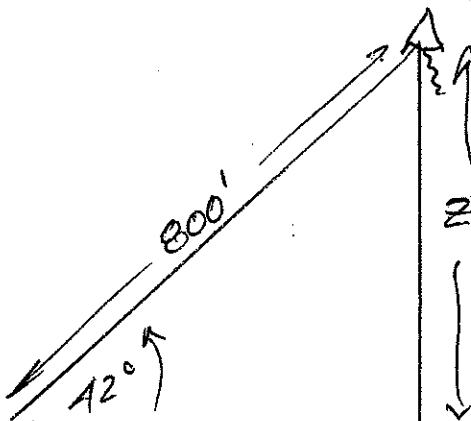


$$\tan 40^\circ = \frac{z}{30}$$

$$z = 30 \tan 40$$

$$z = 25.17 \text{ m}$$

58. A kite string forms an angle of 42 degrees with the ground when the entire 800 feet of string is used. What is the kite's elevation?

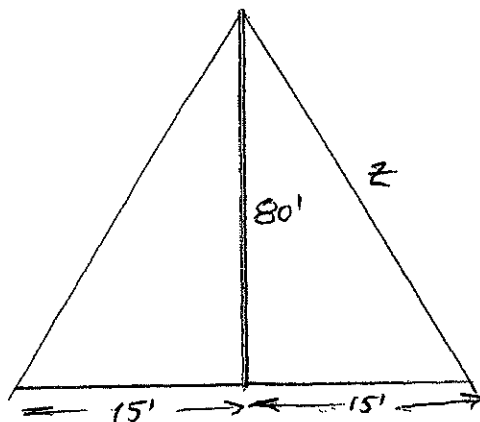


$$\sin 42 = \frac{z}{800}$$

$$z = 800 \sin 42$$

$$z = 535.3 \text{ ft}$$

59. An 80-foot pole is stabilized by guide wires which run from the top of the pole to the ground. The wires are located 15 feet from the base of the pole. What length of wire is required? What is the angle the wire makes with the ground?



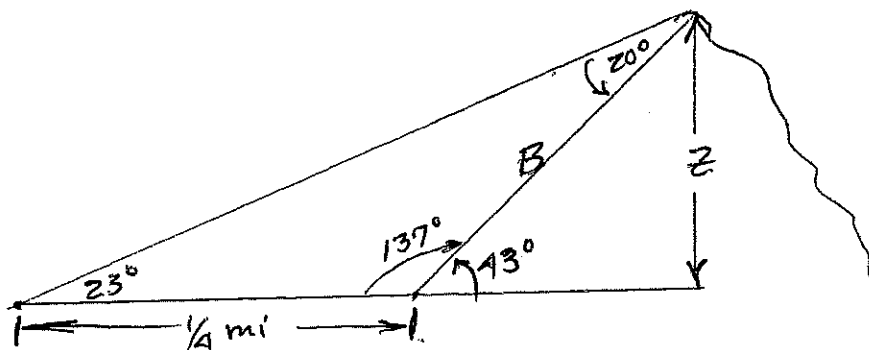
$$z^2 = 15^2 + 80^2$$

$$z = \sqrt{15^2 + 80^2} = 81.4'$$

$$2z = 162.8'$$

$$\theta = \tan^{-1}\left(\frac{80}{15}\right) = 79.4^\circ$$

60. A surveyor measures the angle of elevation of a mountain from point A and finds it to be 23 degrees. The surveyor moves 1/4 mile closer to the mountain and finds the angle of elevation is 43 degrees. What is the height of the mountain?



$$180 - 43 = 137^\circ$$

$$137 + 23 = 160^\circ$$

Law of Sines

$$\frac{0.25}{\sin 20^\circ} = \frac{B}{\sin 23^\circ}$$

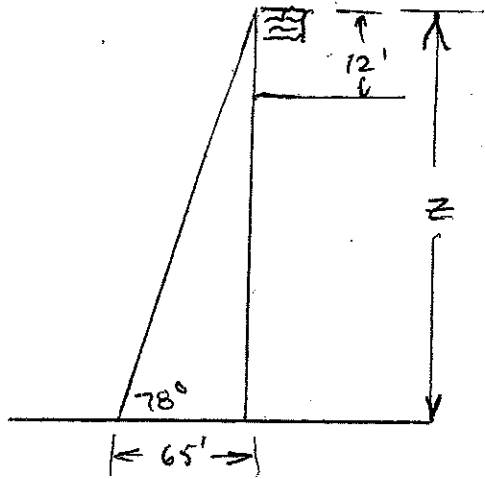
$$B = 0.2856 \text{ mi}$$

$$\sin 43^\circ = \frac{z}{B}$$

$$z = (0.2856) \sin 43^\circ$$

$$z = 0.195 \text{ mi}$$

61. A 12-foot flagpole stands at the edge of a building's roof. The angle of elevation from the ground 65 feet from the building to the top of the flagpole is 78 degrees. Determine the building's height.

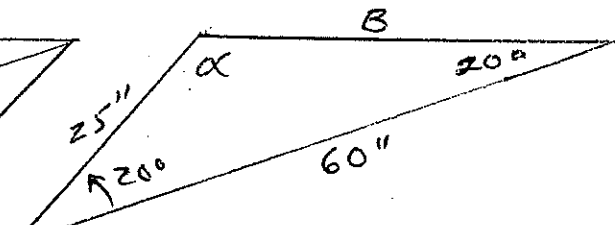
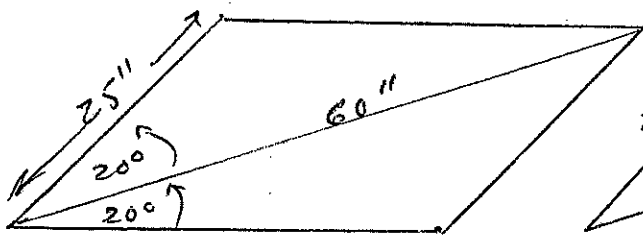


$$\cos 78^\circ = \frac{65}{z}$$

$$z = \frac{65}{\cos 78} = 312.6$$

$$\text{Building height} = 300.6'$$

62. A diagonal of a parallelogram has length of 60 inches and makes an angle of 20 degrees with one of the sides. The side has a length of 25 inches. Determine the length of the other side of the parallelogram.



Diagonal bisects angle

Law of Cosines

$$60^2 = 25^2 + B^2 - 2 \cdot 25 \cdot B \cos \alpha$$

$$\alpha = 180 - 40 = 140^\circ$$

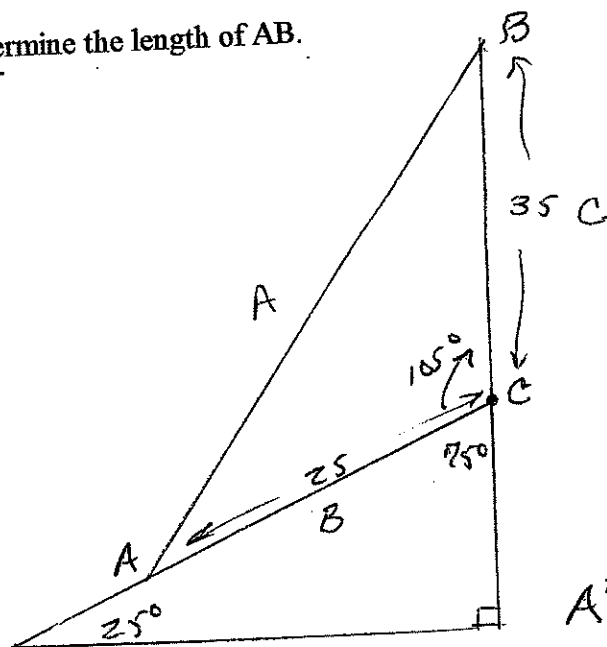
$$3600 = 625 + B^2 - 50B \cos 140$$

$$B^2 + 38.3B - 2975 = 0$$

$$B = \frac{-38.3 \pm \sqrt{38.3^2 - (4)(1)(-2975)}}{2}$$

$$B = \underline{38.67''}, -153.9$$

63. Determine the length of AB.



Law of Cosines

$$A^2 = B^2 + C^2 - 2BC \cos \alpha$$

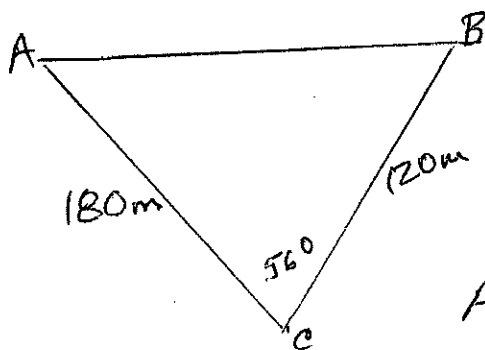
$$A^2 = 25^2 + 35^2 - (2)(25)(35) \cos \alpha$$

$$\alpha = 105^\circ$$

$$A^2 = 625 + 1225 + 412.9$$

$$A = \sqrt{2062.9} \approx 45.41$$

64. A surveyor is determining the distance between two points A and B located on the shoreline. The surveyor is located at point C and measures the distance AC to be 180 m and BC to be 120 m. The angle at C is 56 degrees. Find the distance AB.



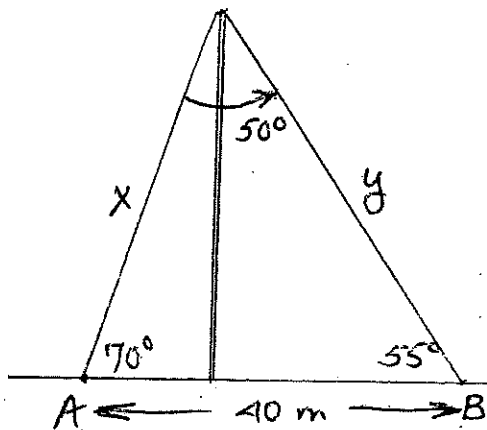
Law of Cosines

$$AB^2 = BC^2 + AC^2 - 2(BC)(AC) \cos 56^\circ$$

$$AB^2 = 120^2 + 180^2 - (2)(180)(120) \cos 56^\circ$$

$$AB = 150.5 \text{ m}$$

65. Two guide wires are attached to the top of a pole and are anchored into the ground on opposite sides of the pole at points A and B. The ground is the same elevation relative to the pole in all directions. The distance AB is 40 m and the angles of elevation at A and B are 70 and 55 degrees respectively. Determine the guy wire lengths.



Law of Sines

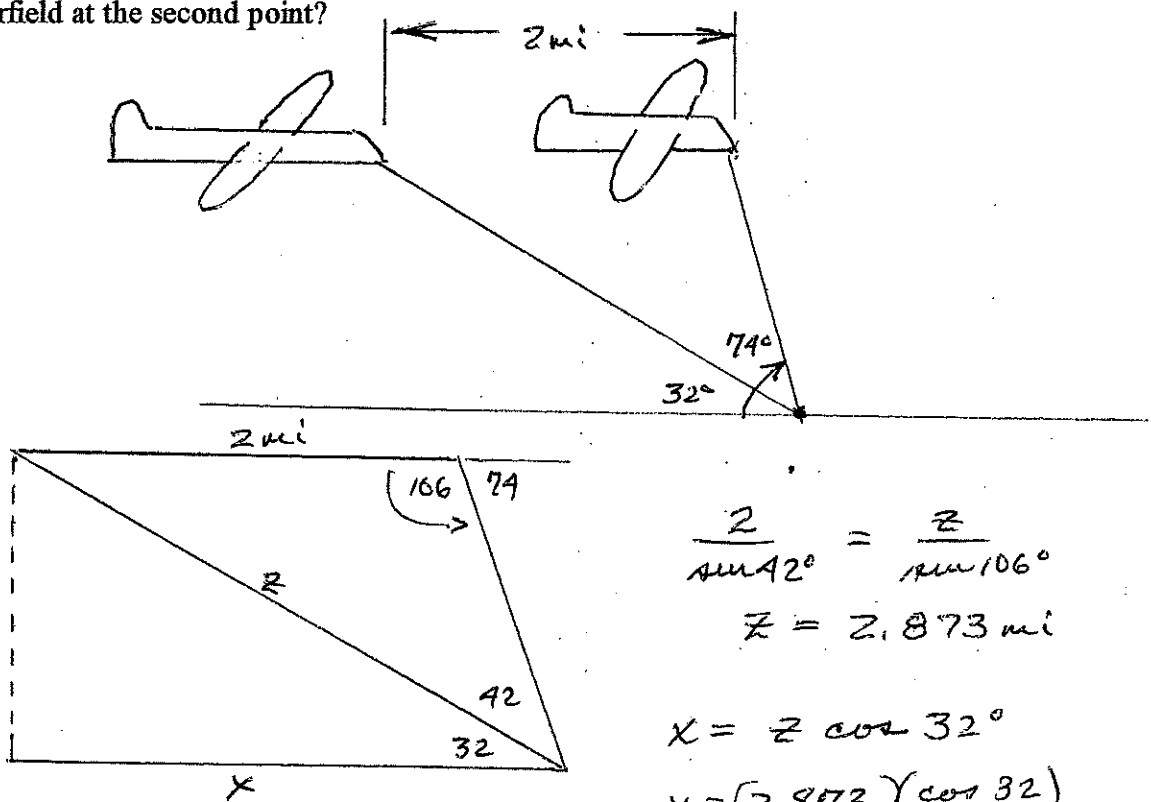
$$\frac{40}{\sin 50^\circ} = \frac{x}{\sin 55^\circ}$$

$$x = 42.77 \text{ m}$$

$$\frac{40}{\sin 50^\circ} = \frac{y}{\sin 70^\circ}$$

$$y = 49.1 \text{ m}$$

67. An airplane is flying in a straight line and at constant elevation towards an airfield. At a given instant the angle of depression between the plane and airfield is 32 degrees. After flying two miles, the angle of depression is 74 degrees. What is the distance between the plane and the airfield at the second point?



$$\frac{2}{\sin 42^\circ} = \frac{z}{\sin 106^\circ}$$

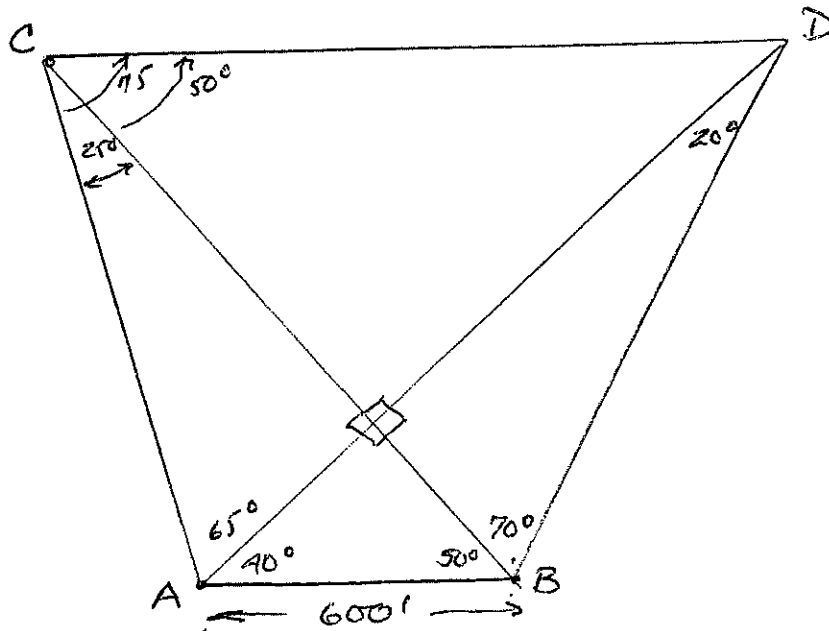
$$z = 2.873 \text{ mi}$$

$$x = z \cos 32^\circ$$

$$x = (2.873)(\cos 32^\circ)$$

$$x = 2.436 \text{ mi}$$

68. In the figure below, points A and B are on the same side of the river where the distance AB is 600 feet. Determine the distance CD on the opposite side of the river.



$$\frac{600}{\sin 20^\circ} = \frac{DB}{\sin 40^\circ}$$

$$DB = 1127.6'$$

$$\frac{CD}{\sin 90^\circ} = \frac{1127.6}{\sin 50^\circ}$$

$$CD = 1472'$$

69. Write the following decimal numbers in powers of ten notation.

- a) 1990      b) 22.22      c) 0.1911

$$1990_{10} = (1 \times 10^3) + (9 \times 10^2) + (9 \times 10^1) + (0 \times 10^0)$$

1000    + 900    + 90    + 0

$$22.22_{10} = (2 \times 10^2) + (2 \times 10^0) + (2 \times 10^{-1}) + (2 \times 10^{-2})$$

20                    2                    0.2                    0.02

$$0.1911_{10} = (1 \times 10^{-1}) + (9 \times 10^{-2}) + (1 \times 10^{-3}) + (1 \times 10^{-4})$$

0.1                    0.09                    0.001                    0.0001

70. Write the following binary numbers in powers of two notation.

- a) 11001      b) 100100111      c) 0.11101

$$11001 = (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

$$100100111 = (1 \times 2^8) + (0 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) \\ + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$

$$0.11101 = (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3}) + (0 \times 2^{-4}) \\ + (1 \times 2^{-5})$$

71. Convert the following decimal numbers to their binary equivalent.

- a) 296      b) 16941      c) 3.1416      d) 0.1110

$$a) (1 \times 2^8) + (0 \times 2^7) + (0 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) \\ \begin{matrix} 256 & + & 0 & + & 0 & + & 32 & + & 0 & + & 8 & + & 0 \\ + & (0 \times 2^1) & + & (0 \times 2^0) \\ + & 0 & + & 0 \end{matrix} = 100101000_2 = 296_{10}$$

$$b) (1 \times 2^{14}) + (0 \times 2^{13}) + (0 \times 2^{12}) + (0 \times 2^{11}) + (0 \times 2^{10}) + (1 \times 2^9) + (0 \times 2^8) \\ \begin{matrix} 16384 & + & 0 & + & 0 & + & 0 & + & 0 & + & 512 & + & 0 \\ + & (0 \times 2^7) & + & (0 \times 2^6) & + & (1 \times 2^5) & + & (0 \times 2^4) & + & (1 \times 2^3) & + & (1 \times 2^2) & + & (0 \times 2^1) \\ + & 0 & + & 0 & + & 32 & + & 0 & + & 8 & + & 4 & + & 0 \\ + & (1 \times 2^0) \\ + & 1 \end{matrix} = 100001000101101_2 = 16941_{10}$$

$$c) (1 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (0 \times 2^{-4}) \\ \begin{matrix} 2 & + & 1 & + & 0 & + & 0 & + & 0.125 & + & 0 \\ + & (0 \times 2^{-5}) & + & (1 \times 2^{-6}) & + & (0 \times 2^{-7}) & + & (0 \times 2^{-8}) & + & (0 \times 2^{-9}) & + & (1 \times 2^{-10}) \\ + & 0 & + & 0.015625 & + & 0 & + & 0 & + & 0 & + & 0.000975 \end{matrix} \\ = 11.0010010001_2 = 3.1416_{10}$$

$$d) (0 \times 2^0) + (0 \times 2^{-1}) + (0 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) + (1 \times 2^{-5}) + (1 \times 2^{-6}) \\ \begin{matrix} 0 & + & 0 & + & 0 & + & 0 & + & 0.0625 & + & 0.03125 & + & 0.015625 \\ + & (0 \times 2^{-7}) & + & (0 \times 2^{-8}) & + & (0 \times 2^{-9}) & + & (1 \times 2^{-10}) & + & (1 \times 2^{-11}) \\ + & 0 & + & 0 & + & 0 & + & 0.000975 & + & 0.000488 \end{matrix} = \\ 0.00011100011_2 = 0.1110_{10}$$

72. Convert the following binary numbers to their decimal equivalent.

- a) 101.101    b) 10001001    c) 0.1110

$$\begin{aligned} \text{a) } & (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \\ & 4 + 0 + 1 + 0.5 + 0 + 0.125 \\ & 101.101_2 = 5.625_{10} \end{aligned}$$

$$\begin{aligned} \text{b) } & (1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) \\ & 128 + 0 + 0 + 0 + 8 + 0 + 0 \\ & + (1 \times 2^0) \\ & + 1 \end{aligned}$$

$$10001001_2 = 137_{10}$$

$$\begin{aligned} \text{c) } & (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3}) + (0 \times 2^{-4}) \\ & 0 + 0.5 + 0.25 + 0.125 + 0 \\ & 0.1110_2 = 0.875_{10} \end{aligned}$$

73. Convert the following decimal numbers to their octal and hexadecimal equivalents.

- a) 19    b) 2015    c) 92.20    d) 0.824

Convert to binary, Then form appropriate groups

$$\begin{aligned} \text{a) } 19_{10} &= 10011_2 \\ 19_{10} &= 23_8 = 13_{16} \end{aligned}$$

$$\begin{aligned} \text{b) } & (1 \times 2^{10}) + (1 \times 2^9) + (1 \times 2^8) + (1 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) \\ & + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \end{aligned}$$

$$2015_{10} = 1111101111_2$$

$$\begin{array}{cccc} & \text{Octal} & & \\ 11 & 111 & 011 & 111 \\ 3 & 7 & 3 & 7 \end{array}$$

$$\begin{array}{ccc} & \text{Hexadecimal} & \\ 0111 & 1101 & 1111 \\ 7 & D & F \end{array}$$



$$c) (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) \\ + (0 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (1 \times 2^{-4})$$

$$92.20_{10} = 1011100.0011$$

Octal				Hexadecimal			
1	011	100.	001 100	0101	1100.	0011	
1	3	4.	1 4	5	C.	3	

$$d) (0 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) + (0 \times 2^{-5}) \\ + (0 \times 2^{-6}) + (1 \times 2^{-7}) + (0 \times 2^{-8}) + (1 \times 2^{-9})$$

$$0.824_{10} = 0.110100101_2$$

Octal				Hexadecimal			
000.	110	100	101	000.	1101	0010	1000
0.	6	4	5	0.	D	2	8

74. Convert the following binary numbers into their hexadecimal equivalent.

a) 1010101101      b) 1011.01      c) 0.101101

a) 0001 0101 0101      155<sub>16</sub>  
       1       5       5

b) 1011.0100      B.4<sub>16</sub>  
       B. 4

c) 0.1011 0100      0.BA<sub>16</sub>  
       0. B 4

75. Add the following pairs of binary numbers.

$$\begin{array}{r} \text{a) } 1001 \\ \underline{101} \end{array}$$

$$\begin{array}{r} \text{b) } 101 \\ \underline{111} \end{array}$$

$$\begin{array}{r} \text{c) } 1101 \\ \underline{10} \end{array}$$

$$\begin{array}{r} \text{d) } 10010010 \\ \underline{10001110} \end{array}$$

$$\begin{array}{r} \text{a) } 1001 \\ \underline{101} \\ 1110 \end{array}$$

$$\begin{array}{r} \text{b) } 101 \\ \underline{111} \\ 1100 \end{array}$$

$$\begin{array}{r} \text{c) } 1101 \\ \underline{10} \\ 1111 \end{array}$$

$$\begin{array}{r} \text{d) } 10010010 \\ \underline{10001110} \\ 10010000 \end{array}$$

76. Subtract the following pairs of binary numbers.

$$\begin{array}{r} \text{a) } 111 \\ \underline{11} \end{array}$$

$$\begin{array}{r} \text{b) } 1001 \\ \underline{111} \end{array}$$

$$\begin{array}{r} \text{c) } 1111 \\ \underline{1001} \end{array}$$

$$\begin{array}{r} \text{d) } 1100011 \\ \underline{1011100} \end{array}$$

$$\begin{array}{r} \text{a) } 111 \\ - 11 \\ \hline 100 \end{array}$$

$$\begin{array}{r} \text{b) } 1001 \\ - 111 \\ \hline 010 \end{array}$$

$$\begin{array}{r} \text{c) } 1111 \\ - 1001 \\ \hline 110 \end{array}$$

$$\begin{array}{r} \text{d) } 1100011 \\ - 1011100 \\ \hline 0000111 \end{array}$$

77. Multiply the following pairs of binary numbers.

$$\begin{array}{r} \text{a) } 1101 \\ \underline{100} \end{array}$$

$$\begin{array}{r} \text{b) } 10000 \\ \underline{10110} \end{array}$$

$$\begin{array}{r} \text{c) } 110 \\ \underline{11} \end{array}$$

$$\begin{array}{r} \text{d) } 110011 \\ \underline{100101} \end{array}$$

$$\begin{array}{r} \text{a) } 1101 \\ \underline{100} \\ 0000 \\ 0000 \\ 1101 \\ \hline 110101 \end{array}$$

$$\begin{array}{r} \text{b) } 10000 \\ \underline{10110} \\ 00000 \\ 10000 \\ 10000 \\ 00000 \\ \underline{10000} \\ 101100000 \end{array}$$

$$\begin{array}{r} \text{c) } 110 \\ \underline{11} \\ 110 \\ 110 \\ \hline 10010 \end{array}$$

$$\begin{array}{r} \text{d) } 110011 \\ \underline{100101} \\ 110011 \\ 000000 \\ 110011 \\ 000000 \\ 000000 \\ 110011 \\ \hline 1110001111 \end{array}$$